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Yakubovsky scheme to study the 4- and 5-nucleon systems in the case of alpha-state structure with spin-dependent nucleon−nucleon potentials

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Abstract

In this project we have investigated the 5-nucleon model system in the picture of the specific alpha-state structure, by extending the Yakubovsky scheme with the inclusion of the spin and isospin degrees of freedom. The Yakubovsky formalism for the 5-nucleon system in the effective alpha-neutron attractive model leads to a set of two coupled equations, based on two relevant alpha-nucleon sub cluster components. To this regard, by switching off the fifth nucleon interactions, the 5-nucleon Yakubovsky equations can be reduced to a typical 4-nucleon problem. To calculate the 5- and 4-nucleon bound state equations, the coupled equations are projected in the momentum space in terms of the Jacobi momenta. In the calculations two different spin-dependent and one spin-independent nucleon−nucleon potential types are dedicated, such as Afnan−Tang S_3, Malfliet−Tjon I/III and Volkov potentials, respectively. The some obtained binding energy differences between the 4-nucleon system in the alpha-state and the 5-nucleon system in the case of alpha-nucleon structure demonstrating the effective interaction between the alpha and an attractive neutron. The obtained results for the effective interaction are consistent either for spin-independent and spin-dependent interactions. Also, the binding energy results are in excellent compatibility with the achieved results by other techniques.

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1. Introduction

Considerable interest has been already shown by both the theorists and the experimentalists in the study of alpha-nucleon interactions, as this process gives insight into nuclear structural problems and also throws some light on the basic two-body interaction. Added interest is also due to the fact that it essentially involves the study of $N = 5$ systems. Incidentally, there is no experimental evidence, so far, for the existence of bound states of five-baryon systems except that of ^{5}He. An objection to the use of simple phenomenological potentials for alpha-nucleon (αN) scattering arises from the fact that such these potentials allow a bound state, for the five-baryon system. In addition, it is well-known the main importance in the few-body problems are finding an exact

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solution for the system, also investigating αN interactions, governing on these systems. Therefore, the investigation of scattering and bound states of nuclei interacting via simple and realistic interactions particularly has been always in the center of the interest and description of light α-core nuclei, especially considering the effective αN interactions, requires well established approaches to the solution of the non-relativistic Schrodinger equation, as well as investigation of the identity of the effective αN interactions.

In recent decades, considerable effort has been made to study the effective αN interaction in the scattering analysis, such as multichannel αN and $\alpha\alpha$ interactions [1], bound-state properties of the ^{6}He and ^{6}Li in a 3-body model, with investigation of αN interactions [2], interactions of αN in an elastic scattering [3], a survey of the αN interaction [4], peripheral αN scattering with NN potential [5] and microscopic calculations of ^{5}He with realistic interactions [6] that are very valuable in determining the extent to which nuclei may be successfully described as a system of nucleons. Beside, significant researches has been exerted to obtain accurate ground-state properties of the nuclear bound systems, particularly for $N \geq 4$ with simple and realistic potentials, such as Stochastic Variational Monte Carlo (SVM) technique [7] which, used simplified with two-body interactions, without realistic nuclear forces and the Nonymmetrized Hyperspherical Harmonics (HH) scheme [8] appears to be pretty promising to deal with permutational-symmetry breaking terms in the Hamiltonian. The HH computational schemes are usually based on the partial-wave (PW) method. The SVM technique, however, is made directly using with vectors in the configuration space. The achievements in [7,8] demonstrate that a direct treatment of the 5-nucleon (5N) systems and beyond is now manageable on the todays computers. Therefore, in order to solve the 5N systems, we suggested that a generalizable and old reliable method, the solution within the Yakubovsky equations, is now desirable. Now, after the study of the four- and six-body bound systems in the case α-core structure within the powerful Yakubovsky scheme, in a typical partial-wave representation [9] and three-dimensional formalism [10] that the technical expertise has been developed and the very strong increase of computational power just recently achieved allows to study the 5N model system problems in that solution of the Yakubovsky equations for the case of alpha-neutron $(\alpha - n)$ model, to estimate the effective 2-body αN interaction. It is worthwhile to mention that a realistic 5N problem is not allowed for a bound state, however, in order to investigate the effective interactions, namely α-particle and an attractive nucleon we make the 5N problem for the case of $\alpha - n$ configuration as a bound system. Also, an objection to the use of simple phenomenological potentials for αN scattering arises from the fact that such these potentials allow a bound state, for the 5N system. Therefore, in order to investigate the effective αN interaction in the specific $\alpha - n$ configuration of the 5N model system, we extend the 5N Yakubovsky equations for ^{5}He, extending the applications to spin-dependent particles and in order to calculate the binding energy results, we evaluate the coupled equations in momentum space on the basis of partial-wave decomposition. Next, we have developed a particular representation of the high-sized eigenvalue matrix, which is methodical with respect to the number of components and well suitable for a numerical implementation. In pursuit of this aim, we investigated the accuracy of the calculations regarding the number of grid points and calculate the expectation value of the 5N Hamiltonian operator.

In this article first, we give a brief review of the 5N Yakubovsky formalism by using the standard cluster notation [11] that leads to four coupled equations with four independent components. Next, we select some specific components, where the $\alpha - n$ approximation is valid and work only with two coupled equations in terms of two remaining components. In order to solve the coupled equations we project these components to the corresponding partial-wave basis states based on Jacobi momenta with the inclusion of spin and isospin degrees of freedom. Next, describes details numerical techniques with typical eigenvalue equation form that characterized the dimension of the problem. Next, we introduce the relevant spin-dependent and spin-independent potential models that are used in the numerical calculations and report the binding energy results for 5N $(\alpha - n)$ and 4N (α-state) problems, with respect to the regarded obtained from other methods. Finally, we evaluate the expectation value energy for testing the accuracy of the numerical implementations and also the concluding remarks are provided.

2. The 5-nucleon Yakubovsky scheme in a sub cluster band

In order to show the specific $\alpha - n$ coupled equations, as well as the anti-symmetrized total state wave function of the effective structure for ^{5}He, a brief review of the Yakubovsky equations to the 5N problem using the sub clusters notation [11] is derived. In view of the expectation for the dominant structure of ^{5}He, namely

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