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# A Free Boundary Problem for pricing a defaultable restricted callable corporate bonds [1]

Jin Liang[a,1], Ying Chen[a]

[a]Department of Mathematics, Tongji University, Shanghai 200092, China

## Abstract

In this paper, a free boundary model for pricing a defaultable and restricted callable corporate bonds is proposed. The pricing model is established and it is turns to a free boundary problem. Numerical analysis and example graphs with optimal calling boundaries are presented.



*Keywords:* defaultable bond; callable bonds; callable restricted condition; free boundary; corporate bond pricing

## 1. Introduction

With the globalization of financial markets and the growing complexity of financial products, among them, bond market is a big one. In the bond market, more and more different conditions are applying the contracts, which active the market and in the same time bring up new risks.

Callable bond contains the terms of the redemption which allows the issuer to redeem all or part of the bond at a given time with the agreed price. By 2011, there were 123 Callable Bonds in China. With the development of the bond market in the country, a variety of bonds which are embedded in single or multiple contingent claims. Among them, callable bonds are a typical one. Especially, some call condition is restricted.

The researches on pricing bonds, such as callable bonds, defaultable bonds can be found in reference [1, 2, 3, 4, 8, 9, 12] etc. However, some defaultable bond with restricted call condition still new to pricing theorem.

In this paper, under the of Black-Scholes framework [6, 7], a model for pricing a defaultable and callable with restricted condition bond is established. The model turns to a the variational inequality problem with restricted condition, i.e. a restricted free boundary problem. In financial meaning, this free boundary is exactly the optimal calling boundary. The numerical analysis and calculations for the model are carried on.

The paper is organized as follows: in the following Section 2, a model is established with assumptions, the model is based on the cash flow of the bond. The model turns to a partial differential equation problem; In Section 3, a free boundary problem is set; Numerical results with optimal calling boundaries are show in Section 4; In Section 5, results are summarized.

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*Corresponding author.
*E-mail address:* liang_jin@tongji.edu.cn.

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## 2. Model

### *2.1. Assumption*

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space. We assume that a firm has a debt including a corporate callable bond , which is a contingent claim of its value on the space $\mathcal{F}$.

**Assumption 2.1 (the corporate bond).** *The firm issue a zero-coupon bond of face value and maturity are F and T, respectively. The discount values of the bonds are denoted by $\varphi_t$ at time t.*

**Assumption 2.2 (the firm asset).** *Let $S_t$ denote the firm's value in the risk neutral world. It satisfies*

$$dS_t = rS_t dt + \sigma S_t dW_t,$$

*where r is the risk free interest rate, and $\sigma$ is volatility of the firm value. They are assumed to be positive constants. $W_t$ is the Brownian motion which generates the filtration $\{\mathcal{F}_t\}$.*

**Assumption 2.3 (coupon).** *Continuous coupon rate h is paid until the call or maturity.*

**Assumption 2.4 (maturity).** *On the maturities of the bonds are agreed that, the investor could receive either the face value of the bounds or all the firm's assets if the firm could not pay the face value i.e.*

$$\varphi_T = \min\{F, S_T\},$$

*if the early call is not happened.*

**Assumption 2.5 (call condition).** *When $S > S_0$, where $S_0$ is a predetermined positive value, the bond issuer has right to call the bond by $F + A$, where $A > 0$.*

*A is so called early calling penalty. If A is too big, the early calling will never happen. So, it is not different to obtain,*

$$A < F(\frac{h}{r} - 1).$$

### *2.2. Cash flow*

The value of $\varphi_t$ is the conditional expectations as follows:

$$\begin{aligned} \varphi(y,t) \quad = \quad & \min_{0<\tau<T, S_\tau > S_0} E_{y,t}\Big[e^{-r(T-t)} \min(S_T, F) \cdot \mathbf{1}_{\tau \geqslant T} + \int_t^{T\wedge\tau} Fhe^{-r(s-t)} ds \\ & + e^{-r(\tau-t)}(F + A)) \cdot \mathbf{1}_{t<\tau<T} \Big| S_t = y\Big], \end{aligned} \tag{2.1}$$

where $\mathbf{1}_{event} = \begin{cases} 1, & \text{if "event" happens ,} \\ 0, & \text{otherwise.} \end{cases}$

### *2.3. PDE problem*

By Feynman-Kac formula and PDE technique (e.x. see [7]), it is not difficult to drive that $\varphi$ is the function of the firm value $S$ and time $t$, which satisfies the following terminate problem of partial differential equation:

$$\begin{aligned} & \frac{\partial \varphi_1}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \varphi_1}{\partial S^2} + rS\frac{\partial \varphi_1}{\partial S} + Fh - r\varphi_1 = 0, \\ & \qquad 0 < S < S_0,\ 0 < t < T, \end{aligned} \tag{2.2}$$

$$\begin{aligned} & \varphi_2 = \min\Big\{\frac{1}{r}\frac{\partial \varphi_2}{\partial t} + \frac{1}{2r}\sigma^2 S^2 \frac{\partial^2 \varphi_2}{\partial S^2} + S\frac{\partial \varphi_2}{\partial S} + Fh, F + A\Big\}, \\ & \qquad S > S_0,\ 0 < t < T. \end{aligned} \tag{2.3}$$