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The Application of Stochastic Bifurcation Theory to the Early Detection of Economic Bubbles

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Abstract

The present research is devoted to the application of stochastic bifurcation theory to the early detection of economic bubbles. A nonlinear random dynamical system with the possible appearance of stochastic P-bifurcations with a fat-tailed probability density function is deduced. The possibility of application of chaotic bifurcation theory to the early detection of culminations of economic bubbles is investigated by the example of dot-com bubbles. For the increments of NASDAQ it is shown that the criterion of reaching the culmination for dot-com bubbles is a formation of a bimodal distribution with the subsequent conversion to a unimodal distribution as a result of codimension one P-bifurcation – a triple equilibrium point.

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Keywords: nonlinear random dynamical system; stochastic bifurcation; economic bubbles; dot-com bubbles; NASDAQ time series.

1. Introduction

The power law of the probability distribution of a signal is one of the distinguishing features for the most of complex systems, regardless of their origin, i.e. a probability density function (PDF) of the signal for $x \to \infty$ has the following form:

$$p(x) \sim x^{-(1+\gamma)}, \gamma > 0. \tag{1}$$

In this case, the complexity of the system is determined by the presence of catastrophic events: unexpected events that cannot be predicted, and extraordinary events that stand out from a series of related events. When describing catastrophic events the PDF (1) is the rule, practically without exception. The fundamental difference of the PDF (1), which belongs to the class of fat-tailed distributions [1], from compact distributions lies in the fact that the events occurring in the tail of the distribution are not rare enough to be neglected [2].

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The PDF (1) describes various kinds of catastrophic events with a good degree of accuracy. Among such events are, for example, dependence of the number of earthquakes on their energy [3]; relative mortality due to natural disasters [4]; a number of cases in epidemics [5]; an area of forest fires [6]; fluctuations in stock indices [7-11]; mass of avalanches [12], etc.

The existence of the PDF (1) is related to the presence of $1/f^{\beta}$ noise (power spectral density is $S(f) \sim 1/f^{\beta}$) in the signal, which is also a criterion of the system complexity. The presence of $1/f^{\beta}$ noise in a system means the possibility of giant fluctuations. This supposes that it stays in the vicinity of the critical point, or the bifurcation point, where such phenomena usually occur. There is a relatively new field in non-linear dynamics – a theory of self-organized criticality [13]. It was created to explain the similar phenomena in systems with the power-series distributions and $1/f^{\beta}$ noises. The existence of the $1/f^{\beta}$ noise in a system means the internal tendency to the catastrophic cases in a system. The theory of self-organized criticality studies the dynamical dissipative systems with the high range of discretion, which operate in the neighborhood of the critical point without the smallest external influence. If the system is in a critical configuration, than small fluctuations can lead to a random event of any "size" with the power-series distribution similar to (1).

If the signal $x(t)$ is the realization of random dynamical system (RDS) with some control parameter α, then stochastic bifurcations [14] may appear in the RDS. In [14] P-bifurcation or phenomenological bifurcation in the presence of noise refers to a qualitative change of $p(x)$ with a small change of control parameter α. For example, a similar bifurcation can lead to the appearance or disappearance of local maxima of the probability distribution for a certain value of the parameter $\alpha = \alpha_P$.

There are a lot of works [14-18] devoted to the study of stochastic bifurcations. However, the number of such studies is noticeably less than the number of publications in the other topical areas of nonlinear dynamics. Moreover, we do not know any papers devoted to the investigation of P-bifurcations in RDS, generating signals with the PDF (1), which is of undoubted interest in connection with the theoretical and practical significance of this distribution.

The practical importance of this study is determined by the possibility of modifying the algorithm for determining the position of the forthcoming bifurcation point and its type from the observed noise change (growth form, saturation level, a density of distribution) [19], taking into account the PDF (1).

2. Stochastic bifurcations in the RDS

First of all, let us define the explicit form of Ito process driven by the standard Wiener process $W(t)$ as an RDS [20]:

$$dx(t) = \mu(x(t),t)\,dt + \sigma(x(t),t)\,dW(t). \tag{2}$$

In the RDS (2) $\mu(x(t),t)$ is a drift coefficient, $D(x(t),t) = \sigma^2(x(t),t)$ is a diffusion coefficient. The Fokker–Planck equation for a PDF of the random variable $x(t)$ is the following equation:

$$\frac{\partial}{\partial t} p(x(t),t) + \frac{\partial}{\partial x} G(x(t),t) = 0, \tag{3}$$

with the following probability current: