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Lie group analysis for bioconvection MHD slip flow and heat transfer of nanofluid over an inclined sheet: Multiple solutions

Ruchika Dhanai, Puneet Rana*, Lokendra Kumar

Department of Mathematics, Jaypee Institute of Information Technology, A-10, Sector-62, Noida 201307, Uttar Pradesh, India

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ABSTRACT

This paper considers numerical analysis of bioconvection boundary layer flow and heat transfer of electrically conducting nanofluid containing nanoparticles and gyrotactic microorganism over an inclined permeable sheet. Lie symmetry group transformations are used to convert the governing non-linear partial differential equations for continuity, momentum, energy, nanoparticles and microorganisms conservation into non-linear ordinary differential equations. The influences of important physical parameters such as mass transfer parameter s, Richardson number Ri, Buoyancy ratio Nr, bioconvection Rayleigh number Rb, velocity and thermal slip parameters, Brownian motion parameter Nb, thermophoresis parameter Nt, the bioconvection Schmidt number Sc_b and the bioconvection Peclet number Pe, on the skin friction, the rate of heat transfer and microorganisms flux are discussed numerically in this study. The dual solutions are obtained for some critical range of mass transfer parameter s and stretching/shrinking parameter χ.

1. Introduction

Heat transfer enhancement is usually a major concern in the many industrial systems and engineering processes. There are various procedures to improve heat transfer efficiency, like changing flow geometry, boundary conditions, or by enhancing thermal conductivity of the fluid. Many techniques are used to enhance heat transfer performance of the basefluid. Some theoretical and experimental studies have reported that the convective heat transfer characteristics can be enhanced by suspending higher thermal conductivity micro- or large sized solid particles into basefluid. However, owing to the large size and high density of the solid particles, settling of particles and clogging in Microsystems were the most common problems. These problems are removed by suspending small sized particles with low concentration and this term is referred as “Nanofluid” by Choi [1] in 1995 at Argonne National Laboratory. Nanofluids are a new class of fluids engineered by suspension of solid particles having diameter between 1 and 100 nm, into a basefluid as water, lubricants and oils, ethylene glycol, biofluids (blood), polymer solutions and other coolants. Nanofluids are generally made of metal (Al, Cu, Ag, Fe), metal oxides (Al_2O_3, CuO, SiO_2, TiO_2), carbides and nitrides (SiC, AlN, SiN) and non-metal (graphite, carbon nanotubes). Nanofluids usually contain up to 5% volume fraction of nanoparticles to ensure heat transfer enhancement. Moreover, the convective transport model for nanofluid has been proposed by Buongiorno [2] and suggested that Brownian and thermophoresis diffusion are the only key mechanisms to develop the superior thermal conductivity over the basefluid. Large numbers of publications show the contribution of researchers in the study of boundary layer flow with utilizing nanoparticle into basefluid. Khan and Pop [3] have studied the boundary layer flow over a stretching sheet utilizing nanoparticles. Later, Rana and Bhargava [4] have extended this work over a nonlinear stretching sheet by incorporating the effect of Brownian diffusion and thermophoresis. Recently, Kuznetsov and Nield [5] have analyzed the boundary layer flow through a vertical plate under the effect of buoyancy forces and concentration of nanoparticles has been controlled passively rather than actively. Magneto-hydrodynamics (MHD) is the study of the complex interaction between the magnetic field and electrically conducting fluids. The various aspects of magnetohydrodynamic boundary layer flow have also attracted many researchers in view of its applications in different mechanical and chemical industries [6–11].Bioconvection is playing an important role in biological systems and biotechnology due to many applications. Various essential processes such as sample pumping, sample separation, sample mixing and sample reaction in biological system, require microfluidic devices [12]. Several techniques have been used to improve mixing efficiency in micromixers. These mixers are categorized by active and passive mixer. Active mixer has better performance over passive mixer but active mixer results higher power consumption and higher fabrication costs. It also generates the Joule heating, which may cause the biological

* Corresponding author. Tel.: +91 9711333514.
E-mail address: puneetranaiitr@gmail.com, puneet.rana1986@gmail.com (P. Rana).

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Nomenclature

a, b	Constant
B	Magnetic field strength (A/m)
C	Nanoparticle volume fraction
C_∞	Ambient volume fraction
D_B	Brownian diffusion coefficient (m^2/s)
D_T	Thermophoretic diffusion coefficient (m^2/s)
D_m	Microorganisms diffusion coefficient (m^2/s)
N	Microorganism concentration
f	Dimensionless stream function
L_1	Velocity slip factor
L_2	Thermal slip factor
N_w	Microorganism concentration at surface
Nr	Buoyancy ratio
M	Dimensionless Magnetic field
Pe	Peclet number
g	Gravitational acceleration
Gr	Grashof number
Nb	Brownian motion parameter
Nt	Thermophoresis parameter
Pr	Prandtl number
Ri	Richardson number
Sc	Schmidt number
s	Mass transfer parameter
Sc_b	Bioconvection Schmidt number
T	Nanofluid temperature (K)
T_w	Nanofluid temperature at surface (K)
T_∞	Ambient temperature (K)
u, v	Velocity components along x- and y-axis (m/s)
w	Dimensionless microorganism concentration
W_c	maximum cell swimming speed
u_w	Velocity of sheet (m/s)
v_w	Mass transfer velocity (m/s)
x, y	Cartesian coordinates (m)

Greek symbol

η, τ	Similarity variable
μ	Dynamic viscosity (Ns/m^2)
ν	Kinematic viscosity (m^2/s)
ϕ	Dimensionless nanoparticle concentration
θ	Dimensionless temperature
Ω	Microorganism concentration difference parameter
φ	Slope of sheet
(ρc)	Heat capacity of nanofluid (J/K)
$(\rho c)_p$	Effective heat capacity of nanoparticle material (J/K)
ρ	Density of nanofluid
β	Thermal expansion coefficient
τ_w	Shear stress at surface (N/m^2)
λ, δ	Velocity and thermal slip parameters
χ	Stretching/shrinking parameter

Subscript

∞	Ambient condition
w	Condition on surface
p	Nanoparticle
nf	Nanofluid

samples to damage during the mixing process. So, this is an important task to decrease the fabrication cost and consumption of power as well as to enhance the mixing efficiency. Bioconvection has wide applications in bio-microsystems, such as enzyme biosensors and biotechnology due to the mass transport enhancement and mixing, which are important issues in many micro-systems [13,14]. Bioconvection has also been contributed to the microfluidic devices like bacteria-powered micromixers and also can be used for developing the stability of nanofluids [15]. Bioconvection has also various applications in the mechanical field where bioconvection can be used for energy source or the mechanical power source after controlling it by applying the electrical field [16,17].

On the other hand, nanofluids are having considerable impact in microfluidic devices such as micro heat pipes, microchannel heat sinks, and microreactors. Thus, the combined study of bioconvection and nanofluid may give more efficient results for microfluidic devices. Bioconvection includes the macroscopic convection of fluid by upswimming microorganisms, those are heavier than water. These self-propelled microorganisms improve the density of the basefluid due to swimming in particular direction. The study of bioconvection of gyrotactic microorganism containing solid particles (nanoparticles) was first studied by Kuznetsov and Avramenko [18] and Geng and Kuznetsov [19]. Kuznetsov [20] has also studied the onset of nanofluid bioconvection in a horizontal layer including the both nanoparticles and gyrotactic microorganisms. Further, Aziz et al. [21] have extended the work of Kuznetsov for boundary layer flow over a horizontal plate in porous medium containing nanoparticles and gyrotactic microorganisms. Kuznetsov [22] has developed the theory for suspension of oxytactic microorganisms and nanoparticles, in which swimming velocity of oxytactic microorganisms is determined by oxygen concentration gradient. Beg et al. [23] have presented the model for laminar flow over a permeable vertical flat wall in a porous medium filled with nanofluid containing oxytactic microorganism. Uddin et al. [24] and Khan et al. [25] have studied the free convection flow of non-Newtonian nanofluid in a porous medium with gyrotactic microorganism. Recently, Xu and Pop [26] have investigated the fully developed mixed convection flow between two parallel horizontal plates filled by nanofluid containing gyrotactic microorganisms. Khan and Makinde [27] have also focused on the study of magnetohydrodynamic bioconvection of nanofluid including gyrotactic microorganisms, past a convectively heated stretching sheet. Lie group analysis is a classical method, also called symmetry analysis was developed by Norwegian mathematician Sophus Lie to find invariant and similarity solutions. This method can successfully be applied to highly nonlinear differential equation. The symmetries of differential equations are those continuous groups of transformations under which the differential equations remain invariant [28]. The symmetry solutions are demanding due to the reduction of the number of independent variables of the problem and used by many researchers intensively [29–37]. The evaluation of multiple solutions for nanofluid flow and heat transfer has gained vast interest among the various researchers. The survey of recent literatures shows that multiple solutions exist for Newtonian and non-Newtonian both type of nanofluid [38–42].

An examination of the literature reveals that no effort is devoted to the study of MHD slip flow and heat transfer analysis of nanofluid containing gyrotactic microorganisms over aninclined stretching/shrinking sheet. Thus, the aim of the present study is to analyze the combined effect of nanoparticles and gyrotactic microorganisms and to capture the multiple solutions of MHD steady boundary layer flow and heat transfer of electrically conducting nanofluid along permeable inclined sheet considering the partial velocity slip and thermal slip conditions at the surface of sheetemploying nanofluid model proposed by Buongiorno [2]. The resulting highly nonlinear system of ordinary differential equations is solved numerically by shooting technique. The influences of controlling parameters on the skin friction, rate of heat transfer and microorganism flux at the surface are discussed numerically and represented graphically in this paper.