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Bridging network static properties and activation dynamics

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Abstract

The paper studies a variant of Granovetter's threshold model of conforming behavior. In this model, a network of agents with binary states (inactive/active) acts in discrete time under the influence of instigators (always active agents). Using special computational method based on algorithms for solving Boolean satisfiability problem, we can find dispositions of small number of instigators among initially inactive agents, which after several time moments force the majority of agents to the active state. Using a large number of networks, both randomly generated, and fragments of real world social networks, we search for the interconnection between static and dynamic characteristics of network vertices: whether or not static centrality measures influence the chances of the vertex to be picked as an instigator.

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1. Introduction

Informally, *conforming behavior* (conformity) is when an individual *conforms* to some group norms. This is a relatively well studied phenomenon in sociology. One of the first mathematical models for conformity was proposed by Mark Granovetter [8]. Essentially, this model considers a group of agents at discrete time moments. Each agent at each time moment can be in one of two states: *active* (1) or *inactive* (0) and decides to be active or inactive at the next time moment based on the observed opinions of other agents. An exact amount, specifying how many active agents there should be for an agent to decide to be active is essentially a *threshold*, individual for each agent. Thresholds are also referred to as *conformity levels*. The extended variant of Granovetter's model, studied, for example, in [4, 11] introduces network structure into the model: a group of agents is considered as a simple directed graph (without loops and multiple arcs), and each agent observes the opinions only of the agents from its neighborhood – those agents from which there are arcs toward a considered agent. Then, naturally, the original Granovetter's model is a special case when the network structure is specified by a complete graph. A generalization of Extended Granovetter's model,

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called Linear Threshold (LT) model [15], uses weighted graphs and a slightly different function to recompute vertices states. In the present paper we focus our study on the Extended Granovetter's model.

Today the study of social networks is a very active direction of research. It gave birth to a number of notions and algorithms that analyze the structure of networks and evaluate the relative importance of some vertices compared to that of another. In particular, for this purposes various *centrality* measures were introduced, such as *degree centrality*, *betweenness centrality*, etc. For a particular vertex the higher the centrality measure is, the more important is this vertex. In the present paper, we want to answer the following question: do 'central' vertices remain central, if we add an additional complexity layer to the network, and introduce dynamics on it? By layer here we mean the conformity levels, and by dynamics – the extended Granovetter's model of conforming behavior. We apply computational approach developed by us in [11, 13] to see, if there is an interconnection between the centrality of a vertex and the likelihood of this vertex to become an always active agent (instigator), which greatly influences this network dynamics. For this purpose we first introduce a heuristic that makes it possible to significantly improve the effectiveness of our computational approach, thus often allowing us to find even better dispositions (of smaller number) of instigators than before. We apply the improved algorithm to analyze several series of networks: both randomly generated according to Barabási-Albert [1] and Erdős-Rényi [6] models, and the fragments of real-world social networks, taken from `Twitter` social networking site.

Let us give a brief outline of the paper. In the next section we provide a description of a studied model of conforming behavior, introduce always active agents (instigators) and related combinatorial problems. Then we outline the main idea of application of Boolean satisfiability problem (SAT) to analysis of considered models. In the third section we propose an idea that makes it possible to significantly improve the practical effectiveness of SAT approach and allows one to find much better dispositions of instigators. In the fourth section we conduct computational experiments in order to answer the question outlined above. After this we consider related works and draw conclusions.

2. Preliminaries

Let us first briefly describe the extended variant of Granovetter's model of conforming behavior that we use throughout the paper and the methods based on algorithms for solving Boolean satisfiability problem that we employ to study it.

2.1. Extended Granovetter's Model of Conforming Behavior

In the extended variant of Granovetter's model [8] a network is represented with a directed graph $G = (V, E)$ without loops and multiple arcs. A set of vertices V, $|V| = n$, corresponds to a set of agents. For each vertex $v_i \in V$ a neighborhood N_i of v_i is defined as a set of vertices from which there is an outgoing arc to v_i: $N_i = \{v_u \in V | (v_u, v_i) \in E\}$. Each vertex is also associated with its conformity level – an integer number c_i, $c_i \in \{1, \ldots, |N_i|\}$.

The binary weights of vertices are synchronously recomputed at discrete time moments $t = 0, 1, \ldots$. The weight of a vertex v_i at the time moment $t = k$ is specified by $w_i^t \in \{0, 1\}$. Then the conforming behavior of agents is defined by the following function.

$$w_i^{t+1} = \begin{cases} 1, \sum\limits_{v_j \in N_i} w_j^t \geq c_i \\ 0, \sum\limits_{v_j \in N_i} w_j^t < c_i \end{cases} \tag{1}$$

The formula means, that, for example, if the conformity level is 1, then the corresponding agent will be active whenever it observes at least one active agent in its neighborhood.

We augment the considered model by introducing *instigators* – agents that are always active. For example, if vertex v_l is an instigator, then $\forall\ t = 0, 1, \ldots$ it holds that $w_l^t = 1$. Instigators make it possible to consider interesting problems regarding activation dynamics in networks, which are covered in more detail in [11]. To distinguish instigators from all the other agents, the non-instigators are referred to as *simple agents*. Hereinafter, by finding a disposition of k instigators we mean that we consider a network, where all agents are simple agents which are inactive at the initial time moment. In this network the goal is to find k simple agents and transform them to instigators to fit our goals. Without the loss of generality, the problem that is repeatedly solved in the paper in different settings is as follows. For