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A decomposition-based multi-objective evolutionary algorithm with quality indicator

Jianping Luo [a,b,*], Yun Yang [a,b], Xia Li [a], Qiqi Liu [a], Minrong Chen [c], Kaizhou Gao [d]

[a] College of Information Engineering, Shenzhen University, Shenzhen 518060, PR China
[b] Shenzhen Key Laboratory of Media Security, Shenzhen 518060, PR China
[c] South China Normal University, Guangzhou 510631, PR China
[d] School of Electrical and Electronic Engineering, Nanyang Technological University, 639798, Singapore

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ABSTRACT

The issue of integrating preference information into multi-objective optimization is considered, and a multi-objective framework based on decomposition and preference information, called indicator-based MOEA/D (IBMOEA/D), is presented in this study to handle the multi-objective optimization problems more effectively. The proposed algorithm uses a decomposition-based strategy for evolving its working population, where each individual represents a subproblem, and utilizes a binary quality indicator-based selection for maintaining the external population. Information obtained from the quality improvement of individuals is used to determine which subproblem should be invested at each generation by a power law distribution probability. Thus, the indicator-based selection and the decomposition strategy can complement each other. Through the experimental tests on seven many-objective optimization problems and one discrete combinatorial optimization problem, the proposed algorithm is revealed to perform better than several state-of-the-art multi-objective evolutionary algorithms. The effectiveness of the proposed algorithm is also analyzed in detail.

1. Introduction

Optimization of multiple conflicting objectives typically arise in the science and engineering fields. Researchers and practitioners formulate such problems as multi-objective optimization problems (MOPs) to minimize or maximize several conflicting objective functions simultaneously. Multi-objective evolutionary algorithms (MOEAs) are widely used to solve MOPs. As MOPs become more complicated, MOEAs encounter many difficulties. The most serious difficulties come from high-dimensional MOPs and diversity maintenance [1–6]. MOEAs contain two conflicting goals: (i) minimizing the distance between approximate Pareto front and the Pareto-optimal front, and (ii) maximizing the diversity within the approximate Pareto front. Most MOEAs, such as NSGA-II [7] and SPEA2 [8], are Pareto-dominance based MOEAs. Such MOEAs exhibit excellent performance on low-dimensional MOPs, but fail in many-objective optimization problems (MaOPs) (i.e., MOPs with more than three objectives). Generating a good approximate Pareto front in these algorithms is not always easy because the rate of non-dominated solutions increases with the number of objectives, which is difficult for the Pareto-based MOEAs to deal with.

The issue of integrating preference information into multi-objective optimization has been addressed by different researchers to handle the MOPs more effectively [9]. The most well-known MOEA based on this strategy is indicator-based evolutionary algorithm (IBEA) [10]. The main idea of IBEA is to formalize preferences in terms of continuous generalizations of the dominance relation, which leads to a simple algorithmic concept. This formalized preference, usually called indicator, can reflect the quality of the solutions in terms of both convergence and diversity. IBEA has attracted the attention of many researchers, and several new MOEAs based on indicators have been developed [11]. In Ref. [12], a multi-objective shuffled frog leaping algorithm based on an indicator was proposed to solve MaOPs. In the present study, a multi-objective framework based on decomposition and preference information, called indicator-based MOEA/D (IBMOEA/D), is proposed. The hypervolume [13] has frequently been used as an indicator function in IBEAs because it has nice features as a performance measure of solution sets in comparison with other measures [14–16,39]. However, it is difficult to be utilized directly because the computation load increases exponentially with the number of objectives. In Ref. [17], an idea of

* Corresponding author. College of Information Engineering, Shenzhen University, Shenzhen 518060, PR China.
E-mail address: ljp@szu.edu.cn (J. Luo).

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approximating the hypervolume using a number of achievement scalarizing functions with uniformly distributed weight vectors was proposed. In Ref. [36], a simple and fast hypervolume indicator-based MOEA (FV-MOEA) is proposed to quickly update the exact hypervolume contributions of different solutions. In Ref. [18], a methodology based on Monte Carlo sampling to estimate the hypervolume contribution of single solutions regarding a specific Pareto set approximation was presented. In this paper, the binary additive ε-indicator [10], which has the properties of simplicity and low computational complexity, is used in IBMOEA/D.

An excellent MOEA based on objective decomposition, called MOEA/D, has recently been proposed by Zhang et al. [19]. Several modified MOEA/D algorithms can be found in the literature [20–22]. In MOEA/D, the MOP is decomposed into a number of scalar optimization subproblems. Different solutions in the current population are associated with different subproblems. The "diversity" among these subproblems will naturally result in diversity in the population. When the decomposition method and the weight vectors are properly selected, such that the optimal solutions to the resultant subproblems are evenly distributed along the *PF*, MOEA/D will have a good chance of search a uniform distribution of Pareto solutions if it optimizes all subproblems well. The original MOEA/D assigns the same computational resources to all the subproblems, however, it may be not the case in certain applications. Suppose that N subproblems are considered in MOEA/D and their weight (or direction) vectors are uniformly distributed to a certain extent. If the MOP is continuous and its *PF* is continuous and convex, then the optimal solutions of these subproblems can constitute a good approximation to the *PF* when the number of weight vectors is sufficiently large. However, in certain cases, such as discrete problems, each subproblem in MOEA/D may have varied contributions to the search process at different search stages [23]. Recently, various combinations of MOEA/D and domination-based techniques have been investigated to address this problem [24–26]. In these combinations, both the decomposition approach and the domination-based approach are used to select good solutions. The main ideas of these studies are that the different subproblems should not be allocated the same amounts of computational resources. The present study attempts to propose a novel mechanism of allocating computational resources to each subproblem dynamically. The main idea is that the more the quality improvement of an individual (subproblem) over several generations is, the more the computational resources assigned to this individual.

Meanwhile, the archive used to save the non-dominated solutions is necessary to improve the final solutions. Many works have been conducted for this research field. Compared with single-objective optimization, where the best solution is always copied into the next population, the incorporation of elitism in MOEAs is substantially more complex. The concept of maintaining an external archive of non-dominated solutions in the evolutionary process is often used. The main objective of the external archive is to keep a historical record of the non-dominated solutions found along the search process. This external archive provides the elitist mechanism for MOEAs. Notably, the archive size is fixed in MOEAs. When the number of non-dominated solutions is more than the maximum size, deleting the inferior non-dominated solutions (i.e., the least useful information), is an important task for the MOEAs. Zitzler et al. [27] showed that elitism helps in achieving better convergence in MOEAs. Among the existing elitist MOEAs, NSGA-II of Deb et al. [7], SPEA2 of Zitzler et al. [8], and Pareto-archived PAES of Knowles et al. [28] have been focused on. In Ref. [23], an external population (EP) management scheme based on NSGA-II selection was also presented. The non-dominated sorting and crowding distance assignment in NSGA-II [7] can effectively select representative solutions of good quality in the case of two or three objectives. However, as discussed previously, NSGA-II selection is ineffective in the case of MaOPs. In the present study, an EP management scheme based on an indicator is used and merged into MOEA/D to store the non-dominated solutions.

Note that our approach differs from literature [11]. Our framework is based on decomposition, and EP based on indicator is used to store the non-dominated solutions in our approach. However, the archive is used to generate new solutions in literature [11]. Also, the proposed algorithm in this paper differs from the literature [23] and [26]. The main differences are as follows: In the External Population (EP), we use an indicator-based approach to evaluate the quality of the individual. It is different from literature [23] in which the method for updating EP is based on NSGA-II selection, and the ways to update the EP are completely different. At the same time, the ways to guide the search of the population are different, we apply a power law distribution probability to select the subproblem to update according to the improvement of the quality of the individual based on the binary additive ε-indicator $I_{\varepsilon+}$ in population, and the EP is not used for guiding. However, in Ref. [23], the EP is used for guiding the search, which is different from our approach.

The major contributions of this work are as follows:

1) A new indicator-based MOEA/D (IBMOEA/D) algorithm, which integrates preference information into multi-objective search, is proposed. The new algorithm is effective to handle MOPs.
2) An archive management scheme based on an indicator is used and merged into MOEA/D to maintain non-dominated solutions found in the evolutionary process. The indicator-based selection, which follows the elitist mechanism, differs from the NSGA-II selection based on non-dominated sorting and crowding distance assignment.
3) The edge effect of the ε-indicator is analyzed in this paper. Meanwhile, we propose an approach to reduce the edge effect.
4) A novel computational resource assignment scheme is developed for each subproblem in the improved MOEA/D. In the original MOEA/D, each subproblem receives the same amount of computational resources. In IBMOEA/D, the likelihood that a subproblem is selected for investment is determined based on its quality evaluated by the indicator.

The remainder of this paper is organized as follows: The second section briefly describes MOPs. The third section describes binary quality indicator. The fourth section introduces our IBMOEA/D algorithm. The fifth section presents the experimental results and further discusses of the algorithm. Finally, the sixth section draws the conclusions.

2. Description of multi-objective problems

An unconstrained m-objective optimization problem can be described by the following equation:

$$\begin{aligned} &Minimize\ \ \mathrm{F(x)} = (f_1(\mathrm{x}), f_2(\mathrm{x}), \ldots, f_m(\mathrm{x}))^T, \\ &\text{Subject to}\ \ \mathrm{x} \in \Omega \end{aligned} \tag{1}$$

where Ω is the decision (variable) space, x is a decision vector, $\mathrm{F}:\Omega \rightarrow \mathrm{R}^m$ consists of m real-valued objective functions, and R^m is the objective space. Thus, we aim to determine the points that yield the optimum