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An aggregation method for solving group multi-criteria decision-making problems with single-valued neutrosophic sets

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ABSTRACT

We develop a novel method that uses single-valued neutrosophic sets (NSs) to handle independent multi-source uncertainty measures affecting the reliability of experts' assessments in group multi-criteria decision-making (GMCDM) problems. NSs are characterized by three independent membership magnitudes (falsity, truth and indeterminacy) and can be employed to model situations characterized by complex uncertainty. In the proposed approach, the neutrosophic indicators are defined to explicitly reflect DMs' credibility (voting power), inconsistencies/errors inherent to the assessing process, and DMs' confidence in their own evaluation abilities. In contrast with most of the existing studies, single-valued NSs are used not only to formalize the uncertainty affecting DMs' priorities, but also to aggregate them into group estimates without the need to define neutrosophic decision matrices or aggregation operators. Group estimates are synthesized into crisp evaluations through a two-step deneutrosophication process that converts (1) single-valued NSs in fuzzy sets (FSs) using the standard Euclidean metric and (2) FSs in representative crisp values using defuzzification. Theoretical and practical implications are discussed to highlight the flexibility of the proposed approach. An illustrative example shows how taking into account the uncertainty inherent to the experts' evaluations may deeply affect the results obtained in a standard fuzzy environment even when dealing with very simple ranking problems.

1. Introduction

Multi-criteria decision-making (MCDM) is a collection of tools and methods used to solve problems with multiple and often conflicting criteria. Group decision-making is the process of making a decision based on feedback from more than one decision maker (DM). Group MCDM (GMCDM) is a complex process involving multiple criteria and multiple DMs. This complexity is amplified when the process involves qualitative and quantitative judgments on the potential alternatives with respect to the relevant criteria. These judgments are often vague and contradictory, and significantly complicate the construction of knowledge-based rules and the establishment of decision support procedures. Vagueness can occur under the following six circumstances: (1) the words that are used in antecedents and consequents of evaluation rules can mean different things to different people [1,2]; (2) consequences obtained by polling a group of experts are often different for the same rule or statement because the experts are not necessarily in agreement [3,4]; (3) decision groups are often heterogeneous due to the different extent of its members' expertise, knowledge and experience [5,6]; (4) expert estimates of criteria importance or performance of alternatives with respect to intangible parameters are not always consistent [7]; (5) information provided by individuals is usually incomplete or ill-defined [8,9]; and, (6) DMs are not always confident about the correctness of their own reasoning [10].

Fundamentally, uncertainty is an attribute of information [11]. There are two main types of uncertainties: external and internal. The *external (or stochastic) uncertainty* implies that the events or statements are well defined, but the state of the system or environmental conditions lying beyond the control of the DM might not be known completely. The *internal uncertainty (or fuzziness)* refers to the vagueness concerning the description of the semantic meaning

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of events, phenomena, or statements themselves, including uncertainties about DM preferences, imprecise judgments and ambiguity of information [12,13]. In this regard, Zadeh [14, p. 28] wrote: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached when precision and significance (relevance) become almost mutually exclusive characteristics." Therefore, precise quantitative analysis is not likely to have much relevance in problems which involve humans either as individuals or in groups.

The presence of multiple vague measures in GMCDM has continued to challenge researchers and the problems associated with finding a comprehensive approach to modeling ambiguous information has still not been adequately resolved. In the sense of Ackoff [15], the problem of having ill-defined goals, ill-defined procedures or ill-defined data is a *mess*. Several theories have emerged during the last 50 years that generalize traditional probability theory and are more appropriate for not-probabilistic information formats in which evidence about uncertainty appears. These include Chiquet's theory of capacities, random set theory, evidence theory, possibility theory, Walley's theory of imprecise probabilities, fuzzy set (FS) theory, rough set theory, intuitionistic fuzzy set (IFS) theory and neutrosophic set (NS) theory, among others [16–21].

The most commonly used methodology for representing and manipulating imprecise and uncertain information in multi-criteria decision systems is the theory of FSs. However, while focusing on the membership grade (i.e., truthfulness or possibility) of vague parameters or events, FSs fail to consider falsity and indeterminacy magnitudes of measured responses. In practical terms, the problem of projecting multi-source and multivariate group decision uncertainty using mathematical models remains intractable in terms of FSs. In the late 90 s Atanassov [17] introduced and developed the idea of IFSs, intuitionistic logic and intuitionistic algebra allowing for more complex mental constructs and semantic uncertainties. In addition to the membership grade, IFSs consider non-membership levels. However, IFSs cannot handle all uncertainty cases, particularly paradoxes. NSs are the cutting-edge concept first introduced by Smarandache [20] in the late 90 s and developed in the 21st century. NSs generalize FSs and IFSs. NSs and, in particular, single-valued NSs are characterized by three independent membership magnitudes, namely, falsity, truth and indeterminacy. Such a formulation allows to model the most general cases of ambiguity, including paradoxes.

1.1. Contribution

This paper proposes a new approach to represent multi-source uncertainty of estimates provided by various domain experts in MCDM problems, and a methodology to integrate these measures within one decision support procedure.

Most of the existing studies on neutrosophic approaches to GMCDM problems focus on the development of aggregation operators to be applied to neutrosophic decision matrices in order to obtain group estimates of criteria and alternatives. At the same time, the truth, falsity and indeterminacy levels used to represent the uncertainty inherent to DMs' judgments are not given an explicit interpretation. These levels are usually treated as abstract triads of (non-standard) reals without highlighting their role as reliability measures or specifying the variables that they depend on.

The artificiality and routineness deriving from an overuse of aggregation operators together with the tendency to overlook a concrete interpretation for neutrosophic values within a given GMCDM problem represent a gap in the literature that need to be consider in order to investigate ways to effectively improve the applicability of decision-making processes.

The proposed GMCDM approach aims at increasing reliability, coherence and dependability of the final outcome by accounting for three different and independent reliability measures that can affect DMs' estimates, namely, DMs' credibility, inconsistency inherent to DMs' evaluation processes, and DMs' confidence in their own evaluation abilities. In order to do so, an assessment procedure for overall priorities of criteria and alternatives is developed using the technology of single-valued NSs.

More precisely, given a committee of M heterogeneous DMs/experts who must evaluate the performance of a set of I alternatives with respect to J criteria, we deal with the following problem.

Problem:

- **Assumption:** Let experts' estimates of the objects (I alternatives and J criteria) be affected by three diverse and independent factors: first, the experts have different credibility (i.e., voting power); second, the local priorities that are derived using relative comparison judgments are characterized by an inconsistency or an error measure; third, due to the lack of information or scarce experience, some experts do not feel confident about their own judgments.
- **Question:** How can these uncertainty metrics be incorporated into a coherent ranking model to increase dependability of the group decision outcome? That is, how can multi-group multi-person expert judgments affected by these uncertainty metrics be coherently formalized and synthesized to yield reliable overall rankings of the criteria and alternatives?

Until recently, modeling and handling independent multi-source uncertainties inherent to a single information unit was challenging due to the lack of appropriate formal tools. With the development of the NS and single-valued NS concepts, the problem of simultaneously handling different ambiguity indicators of one variable can be resolved by converting the values of the indicators into the truth-, falsity- and indeterminacy-membership grades of the corresponding variable. That is, the reliability of any estimate w provided by one of the expert m (i.e., the importance of a criterion or the performance of an alternative with respect to a criterion) can be expressed by a triad of independent magnitudes, $\langle \delta_w^m, \varepsilon_w^m, \theta_w^m \rangle$, where δ_w^m represents the expert's credibility, ε_w^m the inconsistencies/errors intrinsic to the expert's evaluation process and θ_w^m the expert's confidence in his/her own ability and experience to evaluate the importance of the criteria and the performance of the alternatives.

After interpreting triads of reliability measures as neutrosophic values and group estimates as single-valued NSs, a deneutrosophication process is designed to synthesize crisp values representative of group priorities which are, in turn, used to estimate the overall performance of the alternatives.

It must be noted that the proposed formulation assumes independency among the alternatives' performances, i.e., synergy effects do not occur with respect to the alternatives' joint performance. Moreover, non-linear dependencies among criteria, in terms of their importance for the achievement of the overall problem objective, are not considered.

Finally, an illustrative example is provided to show how taking into account multi-source uncertainty indicators inherent to the experts' evaluations may deeply affect the results obtained in a standard fuzzy environment even in the case of very simple ranking problems.

The remainder of this paper is structured as follows. Section 2 offers a literature review focusing the recent applications of FSs, IFSs and NSs to GMCDM problems. Section 3 outlines the key features of the proposed NS-based GMCDM approach highlighting its differences and advantages with respect to the existing models