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A reference direction and entropy based evolutionary algorithm for many-objective optimization

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ABSTRACT

Evolutionary algorithms have been successfully applied in dealing with multi-objective optimization problems with two or three objectives. However, when solving the problem with more than 3 objectives (also called as many-objective optimizing problem), most multi-objective evolutionary algorithms perform poorly due to the ineffectiveness of Pareto dominance relationship in a high-dimensional space, and the diversity maintenance mechanism usually leads the population to be far from the true Pareto front. In this paper, a novel approach is proposed to handle the challenges in the many-objective optimization problem. Firstly, a grid-based approach is adopted to eliminate dominance resistant solutions which are non-dominated solutions with excellent diversity while incur the dominance resistance and lead the population far from the true Pareto front. Secondly, a new diversity maintenance mechanism based on reference directions is proposed, which not only enhances the diversity but also takes the convergence into consideration. For a domination- relationship based MOEA hardly has enough convergence capability for a high-dimension optimizing problem, our approach embeds convergence capability into the diversity maintenance process, and balances the convergence and diversity capability according to evolutionary states and Pareto entropy. The proposed algorithm is evaluated on a number of standard benchmark functions, i.e., DTLZ1-7 and WFG1-9 with 3-, 4-, 5-, 8-, 10-objective and compared with 5 state-of-the-art Many-Objective Evolutionary Algorithms (MaOEAs). Experimental results demonstrate the proposed algorithm's competitiveness in both convergence and diversity in solving Many-Objective Optimization Problems.

1. Introduction

Over the past decades, Multi-Objective Evolutionary Algorithms (MOEAs) have amply shown their optimizing capacity in handling multi-objective problems with two or three conflicting objectives in various applications. They could find a set of well converged and diversified non-dominated solutions in a short time because the algorithms have the ability to reduce the computational complexity tremendously [1–5]. Domination relationship based MOEAs are the most common methods to obtain a Pareto approximation set with well diversity, such as NSGA-II (Non-dominated Sorting Genetic Algorithm-II) [1], MOPSO (Multi-Objective Particle Swarm Optimization) [5] and so on. However, latest researches have demonstrated that these MOEAs fail to solve optimization problems with more than 3 objectives [6–10], which is called Many-Objective Optimization Problems (MaOP). The main reason that domination relationship based MOEAs could not handle MaOP is that the dominated-based ranking methods lose their Pareto optimality and searching capability to compare many- objective solutions. The number of non-dominated solutions dramatically increases with the increase of objective number, which leads to the indiscrimination among those non-dominated solutions. In addition, the number of dominance resistant solutions (DRSs) [7,11,12] which have nearly optimal values in one or two objectives but poor values in the rest objectives, grows as the number of objectives increases. The DRSs are usually well diversified and the crowding distance sorting is unable to eliminate them out of population, but they will incur dominant resistance and deteriorate algorithm's search ability and hinder the population moving toward the true Pareto front. Secondly, a diversity-preservation method (such as the crowding distance sorting) hardly ensures the diversity of population with an increased number of objectives, which is fond of preserving those marginal solutions (e.g. DRSs). Thirdly, a MOEA usually pursues two goals: minimizing the distance to the

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Pareto front (convergence) and maximizing the distribution over the Pareto front (diversity), and adaptively adjusting the search direction between the two goals is very important for the performance of an evolutionary algorithm [13,14]. With the search space being exponentially tremendous, how to balance the convergence and diversity in a many-objective optimization evolutionary algorithm is getting more and more essential. Fourthly, even though not directly related to the optimization ability, it is very difficult for many-objective problems to present a high- dimensional tradeoff front but visualization of a high-dimensional population in the evolution process could help us to exploit population states and facilitate the design of many-objective evolutionary algorithms.

The main contribution of our paper is twofold:

1) A grid-based approach is proposed to eliminate dominance resistant solutions (DRSs) in the first selection level for domination-based evolutionary algorithm. DRSs are non-dominated solutions with excellent values in one or two objectives but with poor values in the others. The number of DRSs grows as the number of objectives increases, and DRSs usually occupy a big proportion of population when using domination relationship based MOEA to solve MaOP. Because DRSs are non-dominated solutions with excellent diversity, the dominated-based ranking and crowding distance sorting will preserve them in the population. But these solutions could incur dominance resistance and take the population away from the true Pareto front, which deteriorates algorithm's convergence. How to eliminate DRSs is seldom investigated and Grid Ranking (GR) based method is applied to handle this issue in this paper. Experiments show that convergence of the algorithm is improved while comparable diversity is maintained with a certain elimination ratio.
2) Two different Euclidean distances to balance convergence and diversity based on the Pareto entropy are proposed in the second level selection. Different from previous works, our approach embeds convergence capacity into the diversity maintenance process. The first level selection of non-dominated ranking could hardly meet the requirement of convergence for MaOP, so the proposed algorithm combines convergence and diversity in the second level selection and balances them based on Pareto entropy. Experiments also demonstrate that introducing convergence capacity into second level selection is necessary for MOEAs solving MaOPs.

The remainder of this paper is organized as follows: Section 2 first discusses the evolutionary methodologies for Many-Objective problems and presents a review of related works on Multi-Objective Evolutionary Algorithm (MOEA) for MaOPs in the next section. In the Section 3, some definitions and notations used in this paper are given, and the framework and details of our proposed Reference Direction and Entropy based Many-Objective Evolutionary Algorithm (RDE-MaOEA) are described in Section 4. Section 5 describes the experimental design and presents empirical results of the proposed approach on benchmark problems compared with 5 state-of-the-art MaOEAs. We conclude the paper with a discussion and a description of future work in Section 6.

2. Related work

Generally speaking, many-objective optimization problem is very similar to multi-objective optimization problem only except with four or more objectives to be optimized simultaneously. However, it brings some new difficulties when the number of optimizing objectives exceeds 3, and the most obvious problem is that the Pareto-optimal front obtained by algorithms could not be visualized by graphical means. It has been discussed in the above section and some previous works [12,15,16] that using the evolutionary algorithm to solve many-objective problems may face following difficulties:

(1) Dominated-based ranking methods lose their Pareto optimality capability to compare many-objective solutions when the number of non-dominated solutions in a population dramatically increases with objective number, which leads to the indiscrimination among those non-dominated solutions.
(2) The domination resistant solutions (DRSs) take a larger proportion of population which will hinder the population toward the true Pareto front.
(3) It is increasingly difficult to balance the convergence and diversity of Pareto solutions in a many-objective optimization evolutionary algorithm.
(4) Visualization is hard to achieve through graphical means.

Although the evolutionary multi-objective algorithms face the above difficulties when they are applied in Many-Objective Optimization Problems (MaOPs), some existing MOEAs are still helpful in finding an acceptable Pareto front for MaOPs through specific techniques. Four common and useful strategies to alleviate the above difficulties will be introduced here.

2.1. A special domination principle

This method uses a special domination principle to push the population toward the true Pareto front [14,17–19]. Grid-based domination techniques have been widely studied and applied in the evolutionary algorithm [14,20]. Knowles and Corne [20] firstly introduced the grid into MOEAs to maintain the diversity of Pareto set. The crowding degree of an individual is calculated by the number of individuals in the same grid location. And the algorithm will select the one with lower crowding degree into the population when two individuals have the same domination rank. A grid-based evolutionary algorithm is proposed in paper [14] for solving Many-Objective Problems. It not only uses the grid to improve the diversity of population, but also introduces grid-based domination to strengthen the selection pressure toward the true Pareto set. Grid ranking (GR) and grid crowding distance (GCD) are introduced to determine the convergence and diversity of individuals in a grid environment. GR is a convergence estimator to rank individuals based on their grid locations, and it aggregates convergence performances of all objectives for an individual. GCD is also calculated by the number of individuals with the same grid location as in [14].

ϵ-MOEA [17] adopts another type of dominance relationship–ϵ-domination to handle problems in the evolution. In the ϵ-domination, the algorithms control the dominance degree by changing the size of ϵ which is used to divide the objective space into hyperboxes, and each hyperbox will be associated with only one individual. The ϵ-domination could also be seen as a grid-based MOEA. Paper [21] developed an adaptive ϵ-dominance based MOEA to address the problem that the boundary individuals may be lost in the evolutionary process.

2.2. Decomposition-based MOEAs

This method adopts a simple idea which decomposes a MaOP into multiple scalar single-objective subproblems and optimizes them simultaneously. Instead of using the Pareto dominance, it generates multiple predefined weight vectors for aggregating all objectives as one and pushes the population to the true Pareto front. Each subproblem is assigned with a weight vector, and it evolves by using only the information from its several nearest neighbors which are measured by the Euclidean distance between their weight vec-