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Flexible job shop scheduling problem with interval grey processing time

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A B S T R A C T

Job processing time is not always given accurately as a time quota in a production system, particularly in the complex products manufacturing process. It is meaningful to study novel model and algorithm based on uncertainty processing time so as to solve uncertainty job shop scheduling problems. This paper defines inaccurate time quota as interval grey processing time and subsequently proposes a novel uncertainty job shop scheduling model with the objective of minimizing the interval grey makespan. By defining the arithmetic operations and Gantt chart of interval grey processing time, the elitism genetic algorithm coupling elitism strategy in external memory is designed. Finally, the proposed algorithm is tested with different size cases of composite components job-shop scheduling. Results show the proposed algorithm is suitable for solving the uncertainty job shop scheduling problem with interval grey processing time.

1. Introduction

Due to the increasing requirements of production renew speed and diversification, job processing time is not always given accurately as a time quota in a production system, i.e. the job processing time is often estimated with low boundary and upper boundary rather than a precise figure. Particularly in the complex products manufacturing process, such as manufacturing scheduling for rockets, planes, complex machines, etc., should be further studied [1]. In the workshop of complex product manufacturing, the most difficult problem of planning management is that job processing time is uncertainty so that traditional scheduling model is invalid [2]. It is obviously valuable to study novel scheduling model with uncertainty information so as to improve production efficiency and level of management.

The scheduling problem of complex product job shop is a flexible job shop scheduling problem (FJSP) proposed by Bruker and Schlie in 1990 [3]. It is an NP-hard problem with the generalization of the classical job shop scheduling problem (JSP) [4]. Many heuristic and meta heuristic algorithms have been adopted to solve the FSJP problem in recent years, such as tabu search [5], simulated annealing [6], genetic algorithm [7], particle swarm optimization [8], ant colony optimization [9], harmony search [10], artificial bee colony [11], artificial immune algorithm [12], imperialist competitive algorithm [13], etc. Especially, the genetic algorithm has gained a lot of attention and been developed in this area [14]. Gao studied genetic algorithm with bottleneck shifting for the FJSP problem [15]. Zhang et al. designed Global Selection (GS) and Local Selection to generate high-quality initial population [16]. In 2012, a modified genetic algorithm with effective selection, crossover and mutation operator was presented [17]. Mokhtari applied GA to solve the FJSP problem considered with energy-efficient multi-objective [18]. And Cinar presented a priority-based genetic algorithm for the FJSP problem in 2017 [19].

Different with traditional FJSP, the processing time is always uncertain rather than in an accurate value. As an indefinite extension of the FJSP problem, the uncertain processing time, due date and so on are mostly represented by fuzzy number [20]. Li studied decomposition-integration and co-evolutionary genetic algorithm for the fuzzy FJSP problem [21]. Considering with fuzzy processing time, an effective teaching-learning-based optimization algorithm was designed by Xu in 2015 [22]. And Gao proposed an improved artificial bee colony algorithm and designed corresponding fuzzy Gantt chart [23]. In 2017, Palacios modeled the uncertain task duration as a fuzzy number and studied a hybrid algorithm to minimize the fuzzy makespan and maximize the robustness [24].

In fact, due to many new products, development pieces of complex products, we can only estimate the approximate range rather

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than the exact distribution of processing time through poor information from employee experience or historical production data. Therefore, the membership function of fuzzy processing time in fuzzy theory is difficult to define. Different from fuzzy theory, grey system theory (GST), proposed by Deng [25], mainly conduct system analysis, decision-making, and optimization based on Grey number [26]. For examples, in Production Management, Liu researched on the production buffer of critical equipment under Grey Information [27]. Zhang applied the grey relational model of grey theory to manufacturing task allocation problem and provided a case study [28]. Fang solved pre-selection of processing equipment under multi-objective conditions by in-depth analysis and grey relational analysis [29]. It is undeniable that Grey number has the features of defined information boundary and uncertain internal law [30]. A grey number is defined as a number that the real value is unknown while the possible values' set could be defined. It is very suitable for estimating processing times of manufacturing tasks. Particularly, the lower and upper boundaries of the possible set could be defined, i.e. processing time is defined as an interval grey number.

However, there are rare studies aimed at the grey number characterization in job shop scheduling. In 2006, Li tried to express the uncertain time parameters in flow shop scheduling with Grey number and adopted grey chance constrained programming to solve the flow shop scheduling problem under uncertainty [31]. But in the process of simulation and computation, the grey number was transformed into a determinate value by the clear weight function. Its information features were not effectively preserved without the direct arithmetic based the grey number information background [32] so that the final Gantt chart could not represent the grey information and practical meaning clearly. Therefore, based on the previous research and practical production background, this paper aims to combine interval grey processing time with the flexible job-shop scheduling problem and design a genetic algorithm to solve this scheduling problem under grey information.

The paper is organized as follows: flexible job shop scheduling problem with interval grey processing time is defined and modeled in Section 2. Operations of interval grey processing time and grey Gantt chart are defined in Section 3. A genetic algorithm was developed based on external memory with elitism strategy in Section 4. Different size cases of composite components job-shop scheduling are designed in Section 5 to test proposed algorithm. Finally, Section 5 concludes the paper.

2. Flexible job shop scheduling with interval grey processing time

A job should be finished in many operations, and different operations may be performed by different work units (machine, individual worker, working group). An operation can be processed by at least one optional work unit in the set of candidate work units. Particularly, the operation processing time could not be defined exactly but can be collected as an interval grey number. Therefore, interval grey processing time should be combined with flexible job shop scheduling problem (FJSP) to solve job shop scheduling problem under uncertainty environment. In this paper, we further define such a model as flexible job-shop scheduling problem with grey processing time (G-FJSP). Similar to traditional FJSP model, G-FJSP model assumed that:

(1) Each job has no priority, i.e. all jobs are equally important.
(2) Each work unit can start the task from the zero moment and has no task at that time. Due to non-stop machines or the shift system for workers, each work unit can be considered to have no rest time.
(3) Once the operation is started, it cannot be interrupted.

According to these assumptions and the definition of traditional FJSP model, Notations of G-FJSP model could be further defined as follows:

$J = \left\{J_i\right\}, i = 1, 2, \cdots, n$ is denoted as a set of n jobs to be scheduled.

$W = \left\{W_k\right\}, k = 1, 2, \cdots, m$ is denoted as a set of all m candidate work units.

$O_{i,h}$ is denoted as one of the operations of the job J_i, where $h \in \left\{1, 2, \cdots q_i\right\}$.

$W\left(O_{i,h}\right) = \{W_e\}, e = 1, 2, \cdots, l_{i,h}$ is denoted as a set of $l_{i,h}$ candidate work units of $O_{i,h}$.

$T^{\otimes}_{i,h,k}$ is denoted as interval grey processing time of $O_{i,h}$ on W_k. It can be shown as an interval grey number $T^{\otimes}_{i,h,k} \Rightarrow T^{*}_{i,h,k} \in \left[T^{-}_{i,h,k}, T^{+}_{i,h,k}\right]$ where $i = 1, 2, \cdots, n;\ h = 1, 2, \cdots, q_i;\ k \in W\left(O_{i,h}\right)$, $a_{i,h,k}, b_{i,h,k} \in \mathbf{R}^{+}$, $T^{-}_{i,h,k} < T^{+}_{i,h,k}$. Interval grey number $T^{\otimes}_{i,h,k}$ means the possible processing time $T^{*}_{i,h,k}$ is inside the interval value set $\left[T^{-}_{i,h,k}, T^{+}_{i,h,k}\right]$, i.e. from $T^{-}_{i,h,k}$ to $T^{+}_{i,h,k}$.

$S^{\otimes}_{i,h}$ is denoted as the interval grey processing starting time of $O_{i,h}$. Similarly, it can be defined as $S^{\otimes}_{i,h} \Rightarrow S^{*}_{i,h} \in \left[S^{-}_{i,h}, S^{+}_{i,h}\right]$.

$E^{\otimes}_{i,h}$ is denoted as interval grey processing ending time of $O_{i,h}$. Thus $E^{\otimes}_{i,q_i}$ denotes the completed time of J_i and $T^{\otimes}_{i,h,k}$: $E^{\otimes}_{i,h} \Rightarrow E^{*}_{i,h} \in \left[E^{-}_{i,h}, E^{+}_{i,h}\right]$.

$I^{\otimes}_{i,h}$ is denoted as the grey interval time between $O_{i,h}$ and $O_{i,h-1}$, and $I^{\otimes}_{i,h} \Rightarrow I^{*}_{i,h} \in \left[I^{-}_{i,h}, I^{+}_{i,h}\right]$. Particularly, $I^{\otimes}_{i,1} = 0$.

$C^{\otimes}_{max}$ is denoted as the maximum grey completed time of all jobs, $C^{\otimes}_{max} = \max\limits_{1 \le i \le n}\left(E^{\otimes}_{i,q_i}\right)$.

L is a maximal positive number.

$$x_{i,h,k} = \begin{cases} 1, \text{if} O_{i,h} \text{is executed by} W_k \\ 0, \text{otherwise} \end{cases}.$$

$$y_{i,h,j,p,k} = \begin{cases} 1, \ \text{if} O_{i,h} \text{ is before} O_{j,p} \text{ on} W_k \\ 0, \ \text{otherwise} \end{cases}.$$

In order to complete the jobs as much as possible in a period of time, job-shop scheduling mainly concerned with the completion time of all jobs rather than total work unit workload, job delivery time, each work unit workload, work-in-process inventory, etc. Therefore, the primary objective of this problem is to minimize makespan of all jobs. Therefore, the mathematical model can be established as follows.

$$\text{Min} C^{\otimes}_{max} = \max_{1 \le i \le n}\left(E^{\otimes}_{i,q_i}\right)$$
$$s.t.\begin{cases} S^{\otimes}_{i,h} + x_{i,h,k} \times T^{\otimes}_{i,h,k} \le E^{\otimes}_{i,h} & (1.1) \\ E^{\otimes}_{i,h} + I^{\otimes}_{i,h+1} \le S^{\otimes}_{i,h+1} & (1.2) \\ S^{\otimes}_{i,h} + T^{\otimes}_{i,h,k} \le S^{\otimes}_{j,p} + L\left(1 - y_{i,h,j,p,k}\right) & (1.3) \\ \sum_{k=1}^{l_{i,h}} x_{i,h,k} = 1 & (1.4) \\ S^{\otimes}_{i,h} \ge 0 & (1.5) \\ i = 1, 2, \cdots, n; j = 1, 2, \cdots, n & \\ h = 1, 2, \cdots, q_i - 1; p = 1, 2, \cdots, q_j & \\ k \in W\left(O_{i,h}\right) & \end{cases} \quad \text{(Model I)}$$

Formula (1.1) and (1.2) represent the sequence constraint of each job: the end time of the process can't be earlier than the start