Applied Soft Computing 65 (2018) 349–359

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

A group decision making model based on triangular fuzzy additive reciprocal matrices with additive approximation-consistency

Fang Liu*, Zu-Lin Liu, Yu-Hao Wu

School of Mathematics and Information Science, Guangxi University, Nanning Guangxi 530004, China

ARTICLE INFO

Article history:
Received 1 February 2017
Received in revised form 16 January 2018
Accepted 16 January 2018

Keywords:
Group decision making (GDM)
Triangular fuzzy number
Additive approximation-consistency
Induced ordered weighted averaging (IOWA) operator
Compatibility degree

ABSTRACT

A group decision making (GDM) model is proposed when the experts evaluate their opinions through triangular fuzzy numbers. First, it is pointed out that the preference relations with triangular fuzzy numbers are inconsistent in nature. In order to distinguish the typical consistency, the concept of additive approximation-consistency is proposed for triangular fuzzy additive reciprocal matrices. The properties of triangular fuzzy additive reciprocal matrices with additive approximation-consistency are studied in detail. Second, using $(n-1)$ restricted preference values, a triangular fuzzy additive reciprocal preference relation with additive approximation-consistency is constructed. Third, a novel compatibility degree among triangular fuzzy additive reciprocal preference relations is defined. It is further applied to introduce the compatibility-degree induced ordered weighted averaging (CD-IOWA) operator for generating a collective triangular fuzzy additive reciprocal matrix with additive approximation-consistency. Finally, a new algorithm for the group decision-making problem with triangular fuzzy additive reciprocal preference relations is presented. A numerical example is carried out to illustrate the proposed definitions and algorithm.

1. Introduction

Owing to the increasing complexity of socio-economic environments, it is less and less possible for an expert to consider all relevant aspects of a decision making problem. Therefore, many organizations employ multiple experts to reach a decision, which is called as group decision making (GDM). To model GDM problems, all the experts may evaluate their judgements by using preference representation formats, such as fuzzy preference relations [1–3], multiplicative preference relations [4–6], linguistic frame work [7,8] and so on.

It is noted that a precise numerical value can not reflect the incomplete and vague knowledge of the expert's preference level. To model uncertainty in practical problems, various theories have been proposed such as the probability theory and the possibility theory [9,10]. In the context of possibility theory, the fuzzy set theory can be utilized to rationalize uncertainty associated with vagueness in a manner analogous to human thought [11,12]. The vagueness experienced by decision makers can be quantified as possibility distributions in terms of set memberships and further processed such as reasoning and optimizing by virtue of an evolutionary algorithm [5,13,14]. These set memberships include interval numbers, triangular fuzzy numbers, trapezoidal fuzzy numbers, L-R fuzzy numbers and others [15]. Some of the above fuzzy formats have been conveniently assumed to be utilized by the experts in a GDM process [16]. In order to distinguish two cases of reciprocal properties, the terminologies of additive and multiplicative reciprocal matrices have been proposed [17]. Then different preference formats have been given such as interval additive reciprocal preference relations [18,19], interval multiplicative reciprocal preference relations [20,21], triangular fuzzy multiplicative reciprocal preference relations [22,23], trapezoidal fuzzy multiplicative reciprocal preference relations [24] and others. Moreover, in investigating decision making problems, one of the important issues is the consistency of preference relations [25,26] in order to avoid self-contradiction of decision makers. It has attracted much attention and many definitions of consistent preference relations have been presented. For example, there are consistent multiplicative reciprocal preference relations [4], additive reciprocal preference relations with multiplicative and additive consistency [1,27], consistent interval multiplicative

* Corresponding author.
E-mail addresses: fang272@126.com (F. Liu), 2872695218@qq.com (Z.-L. Liu), 1377622108@qq.com (Y.-H. Wu).

https://doi.org/10.1016/j.asoc.2018.01.020
1568-4946/

reciprocal preference relations [28,29], interval additive reciprocal preference relations with multiplicative and additive consistency [30–32], consistent triangular fuzzy multiplicative reciprocal preference relations [33,34] and triangular fuzzy additive reciprocal preference relations with multiplicative consistency [35,36]. It is seen that the above consistency definitions of preference relations are based on the typical idea of consistency. Recently, by comparing the ideas of fuzzy sets and typical consistency, it is pointed out that interval-valued matrices are inconsistent in nature and the novel concept of approximate consistency has been proposed in [37,38]. Motivated by the idea in [37], one can see that the preference relations with triangular fuzzy numbers are also inconsistent in nature [39]. It is very interesting and important to define the approximate consistency of triangular fuzzy additive reciprocal preference relation, then to address its application in a GDM problem.

The objective of this paper is to present a novel model for GDM with triangular fuzzy additive reciprocal matrices. The main novelty is to give the new concept of additive approximation-consistency of triangular fuzzy additive reciprocal matrices. The randomness experienced by decision makers in pairwisely comparing the alternatives and the additive reciprocity of preference relations are considered. Furthermore, from $(n-1)$ restricted triangular fuzzy preference values, the triangular fuzzy additive reciprocal preference relation with additive approximation-consistency is constructed. A novel IOWA operator is presented to aggregate triangular fuzzy additive reciprocal preference relations based on the compatibility degrees. The aggregation of the decision makers' preference relations follows the idea that the more importance is given to that with the more compatibility degree. In what follows, we give the structure of this paper. Section 2 introduces the preliminaries. In Section 3, a new definition of triangular fuzzy additive reciprocal preference relations with additive approximation-consistency is proposed and the properties are studied in detail. Section 4 shows a new method of constructing triangular fuzzy additive reciprocal preference relations using only $(n-1)$ restricted preference values. The CD-IOWA operator is proposed for the aggregation of individual triangular fuzzy additive reciprocal matrices. In Section 5, a new algorithm is presented for the GDM problems with triangular fuzzy additive reciprocal matrices. A numerical example is offered to illustrate the proposed definitions and methods. The main conclusions are covered in Section 6.

2. Preliminaries

It is considered that a GDM problem is with a finite set of alternatives $X=\{x_1, x_2, \ldots, x_n\} (n \geq 2)$. The alternatives would be ranked from the best to the worst by making use of preference relations provided by a group of experts $E=\{e_1, e_2, \ldots, e_m\} (m \geq 2)$. Each expert e_k compare every pair of alternatives to give a preference value in decision making processes according to the analytic hierarchy process (AHP) [4]. In what follows, prior to giving the definition of triangular fuzzy additive reciprocal preference relations, let us firstly recall the definition of additive reciprocal preference relations [1,40]. An additive reciprocal preference relation B on a set of alternatives X is a fuzzy subset on the product set $X \times X$, which is characterized by a membership function

$$\mu_b : X \times X \to [0, 1].$$

The preference relation may be conveniently expressed as the matrix $B=(b_{ij})_{n\times n}$, where b_{ij} is represented by $b_{ij}=\mu_b(x_i, x_j)$ and it is interpreted as the preference ratio of the alternative x_i over x_j, for all $i, j=1, 2, \ldots, n$. For example, $b_{ij}=1/2$ indicates that there is no difference between x_i and x_j $(x_i \sim x_j)$; $b_{ij}=1$ means that x_i is absolutely preferred to x_j, and $1/2<b_{ij}<1$ implies that x_i is preferred to $x_j (x_i \succ x_j)$. Furthermore, the preference relation B is typically assumed to have the additive reciprocity as follows:

$$b_{ij} + b_{ji} = 1, \quad \forall i, j = 1, 2, \ldots, n.$$

Moreover, to make a reasonable decision, a decision maker should know how to define, check and improve the consistency of preference relations. Consequently, the consistency properties of additive reciprocal preference relations have been studied comprehensively [1]. Those properties include the triangle condition, the weak transitivity, the max–max transitivity, the restricted max–min transitivity, the restricted max–max transitivity, and the additive transitivity. The additive transitivity of additive reciprocal preference relations can be considered as the equivalent concept of Saaty's consistency of multiplicative reciprocal preference relations [41]. As shown in the following definition, an additive reciprocal preference relation with additive transitivity is called as an additive preference relation with additive consistency [1,27].

Definition 1. An additive reciprocal preference relation $B=(b_{ij})_{n\times n}$ is considered to be additively consistent, if it satisfies the following additive transitivity

$$b_{ij} = b_{ik} + b_{kj} - 0.5, \quad \forall i, j, k = 1, 2, \ldots, n.$$

It is seen that in modeling GDM problems by additive reciprocal preference relations, all the experts are required to provide crisp preference ratios. The requirement is always impossible to meet for the reason that the experts may have some difficulties to evaluate their opinions using real numbers due to the complexity and vagueness involved in real world decision problems and their incomplete information or knowledge. To rationalize uncertainty associated with vagueness, the fuzzy set theory is feasible [11]. Triangular fuzzy number is one of the major components and it is more natural than a precise value in simulating uncertainty associated with vagueness. For example, in a real situation such as comparing projects with respect to criteria, the decision maker may lack information about the considered problem. Then her/his incomplete understanding of the problem leads to the vagueness about the estimation of the preference intensity of x_i over x_j. To quantify the vagueness, the expert faithfully expresses the lower point of the preference intensity, the upper point, and the most probable point respectively [16]. It means that the expert proposes a triangular fuzzy number to evaluate her/his judgements [22,23]. A fuzzy number Q on $\Re$ is said to be a triangular fuzzy number if its membership function $Q(x) : \Re \to [0, 1]$ is equal to

$$Q(x) = \begin{cases} \dfrac{x-l}{m-l}, & l \leq x \leq m \\ \dfrac{u-x}{u-m}, & m \leq x \leq u \\ 0, & otherwise \end{cases}, \tag{1}$$

where l and u represent the lower and upper bounds of the fuzzy number Q, respectively, and m is the median value. The triangular fuzzy number is shown in Fig. 1 and denoted as $Q=(l, m, u)$ for the sake of simplicity.

In addition, letting $Q_1=(l_1, m_1, u_1)$ and $Q_2=(l_2, m_2, u_2)$ be two triangular fuzzy numbers, one has the following simplified operation laws [12]:

- Triangular fuzzy number addition $\oplus$:

$$Q_1 \oplus Q_2 = (l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2). \tag{2}$$