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Group decision making based on incomplete multiplicative and fuzzy preference relations

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ABSTRACT

The main aim of this paper is to investigate the group decision making on incomplete multiplicative and fuzzy preference relations without the requirement of satisfying reciprocity property. This paper introduces a new characterization of the multiplicative consistency condition, based on which a method to estimate unknown preference values in an incomplete multiplicative preference relation is proposed. Apart from the multiplicative consistency property among three known preference values, the method proposed also takes the multiplicative consistency property among more than three values into account. In addition, two models for group decision making with incomplete multiplicative preference relations and incomplete fuzzy preference relations are presented, respectively. Some properties of the collective preference relation are further discussed. Numerical examples are provided to make a discussion and comparison with other similar methods.

1. Introduction

Preference relations (PRs) are commonly used methods to express the preference information of decision makers (DMs) in decision making. In the past decades, numerous studies on this issue have been performed and various types of PRs [11,15,16,33,40–42] have been introduced in succession, among which multiplicative preference relations (MPRs) and fuzzy preference relations (FPRs) received much research attention [8–10,12–14,21,22,26,32,34–38,44,45,52–57]. In MPRs and FPRs based decision making process, DMs should provide their preferences by means of evaluations over each pair of alternatives and construct the corresponding judgement matrices.

For constructing such judgement matrices, the DMs have to give a preference degree of one alternative over another when comparing each pair of alternatives and $n(n$-1) times of judgements are required if a complete PR with n alternatives is constructed. However, due to the complexity and uncertainty of real world, time pressure or not possessing a sufficient level of knowledge of part of the problem [1,2,46–48], DMs sometimes may have difficulty providing complete judgments. As a result, providing incomplete PRs with some missing or unknown preference values become a realistic choice to express DMs' preferences. Up to now, a series of models and methods with incomplete PRs have been developed, specially on how to estimate the unknown preference values and obtain the priority vectors. For example, Alonso et al. [3] proposed a procedure for finding out the missing information in an expert's incomplete FPR based on additive consistency. Herrera-Viedma et al. [20] proposed an iterative procedure to estimate the missing information in an expert's incomplete FPR based on additive consistency property. Alonso et al. [1] put forward a general procedure for estimating the missing information of incomplete PRs with several formats. Herrera-Viedma et al. [19] presented a characterization of the consistency property defined by the additive or multiplicative transitivity property of the FPRs. Lee [23] proposed a method for estimating unknown preference values based on the additive consistency. Chen et al. [6] discussed the drawbacks of Lee's method and presented an improved method for group decision making (GDM) using incomplete FPRs. Xu [49] defined the concepts of incomplete FPRs, additive consistent incomplete FPRs and multiplicative consistent incomplete FPRs, and then proposed two goal programming models for obtaining the priority vectors for incomplete FPRs. Gong [18] developed a least-square model for obtaining the collective priority vectors for incomplete PRs. Xu and Chen [50] developed a simple method for deriving the ranking of the alternatives from an incomplete reciprocal relation based on additive transitivity. Xu et al. [43] gave a definition of multiplicative consistent for incomplete FPRs and extended the logarithmic least squares method for deriving priorities from group incomplete FPRs. Liu et al. [24] proposed a method for determining the priority weights of FPRs, and presented a least square completion and inconsistency repair methods for dealing with incomplete and inconsistent FPRs.

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Most of the existing literature about incomplete PRs has two prominent characteristics: (1) Many studies are based on the basic assumption that the PRs satisfy reciprocity properties, that is, FPRs and MPRs are additive and multiplicative reciprocal, respectively. (2) The estimating of unknown values of incomplete PRs is mainly based on an iterative procedure proposed by Alonso et al. [1], where three cases of the multiplicative or additive consistency property among three elements of incomplete PRs under are considered.

However, in real decision making situations, it is not uncommon that preferences provided by DMs do not fully comply with any transitivity or even reciprocity properties [4,5]. And the existing models and methods on reciprocal FPRs and MPRs are not suitable for solving decision making problems with FPRs and MPRs not satisfying reciprocity property. For example, given a MPR not satisfying reciprocity property, the weight vector derived by geometric mean method denoted by (12) can not preserve the original information of the given MPR as much as possible. In addition, more than three cases of the consistency property among elements of incomplete PRs can be applied to estimate its unknown values. Based on these considerations, this paper develops new decision making models with incomplete MPRs and FPRs without the requirement of satisfying reciprocity property. The idea of the method reflects as follows: a procedure to determine unknown preference values in incomplete PRs not satisfying reciprocity property is developed based on more comprehensive multiplicative consistency given by Proposition 3.1 and the corresponding method to obtain the weight vector is presented, which preserves the information of the complete PR as much as possible. As a result, fewer times of iteration is needed to estimate the unknown values if the incomplete PRs are acceptable [51], i.e., all of the unknown preference values can be estimated, and the corresponding complete PRs derived may be of higher consistency level.

The rest of this paper is structured as follows. In Section 2, a brief introduction to the basic notions is provided. Section 3 describes some Propositions on complete MPRs, based on which a method for estimating unknown preference values in an incomplete PR is proposed. In Section 4, two models for GDM with incomplete MPRs and FPRs are presented, respectively. In Section 5, the comparison with other similar methods is provided. Section 6 gives the conclusions.

2. Preliminaries

For simplicity, let $X = (x_1, x_2, \ldots, x_n)$ be a finite set of alternatives and $I = \{1, 2, \ldots, n\}$ be the set of index [7,17,30].

Definition 1. A FPR $P = (p_{ij})_{n\times n}$ on the set X is a fuzzy set on the product set $X \times X$, which is characterized by a membership function $\mu_P : X \times X \to [0, 1]$, where p_{ij} denotes the preference degree of the alternative x_i over x_j, $p_{ii} = 0.5$, $1 \le i \le n$. Specially, $p_{ij} = 0$ indicates that x_j is absolutely preferred to x_i; $p_{ij} = 0.5$ indicates indifference between x_i and x_j; $p_{ij} > 0.5$ indicates that x_i is preferred to x_j; $p_{ij} = 1$ indicates that x_i is absolutely preferred to x_j.

P is called an additive reciprocal FPR if the following condition is satisfied [31]:

$$p_{ij} + p_{ji} = 1,\ \forall \mathrm{i}, \mathrm{j} \in \mathrm{I}. \tag{1}$$

Definition 2. An additive reciprocal FPR $P = (p_{ij})_{n\times n}$ is additively consistent, if the following additive transitivity is satisfied

$$p_{ij} = p_{ik} - p_{jk} + 0.5,\ \forall \mathrm{i}, \mathrm{j}, \mathrm{k} \in \mathrm{I}. \tag{2}$$

And $P = (p_{ij})_{n\times n}$ is multiplicatively consistent, if the following multiplicative transitivity is satisfied

$$\frac{p_{ji}p_{kj}}{p_{ij}p_{jk}} = \frac{p_{ki}}{p_{ik}},\ \forall \mathrm{i}, \mathrm{j}, \mathrm{k} \in \mathrm{I}. \tag{3}$$

Note that Eq. (3) is given based on the assumptions: $p_{ij} \ne 0$ and $p_{ij} \ne 1, \forall i, j \in I$ [27,28].

Definition 3. A MPR A on a set of alternatives X is represented by a matrix $A \subset X \times X$, $A = (a_{ij})_{n\times n}$, where a_{ij} is the preference ratio of alternative x_i over x_j, $a_{ij} > 0$, $a_{ii} = 1$, $\forall i, j \in I$. Specially, $a_{ij} < 1$ indicates that x_j is preferred to x_i; $a_{ij} = 1$ indicates indifference between x_i and x_j; $a_{ij} > 1$ indicates that x_i is preferred to x_j.

A is called a reciprocal MPR if the following condition is satisfied [29]:

$$a_{ij}a_{ji} = 1,\ \forall \mathrm{i}, \mathrm{j} \in \mathrm{I}. \tag{4}$$

Definition 4. A reciprocal MPR $A = (a_{ij})_{n\times n}$ is called a consistent MPR, if the following multiplicative transitivity is satisfied

$$a_{ij} = a_{ik}a_{kj},\ \forall \mathrm{i}, \mathrm{j}, \mathrm{k} \in \mathrm{I}. \tag{5}$$

Herrera-Viedma et al. [19] gave the following characterization of multiplicative consistency.

Proposition 2.1. For a reciprocal MPR $A = (a_{ij})_{n\times n}$, the following statements are equivalent:

(i) $a_{ij}a_{jk} = a_{ik}\ \forall i, j, k$.
(ii) $a_{ij}a_{jk} = a_{ik}\ \forall i < j < k$.
(iii) $a_{ij} = a_{ii+1}a_{i+1i+2}\ldots a_{j-1j}\ \forall i < j$.

3. Determine unknown preference values in incomplete PRs

In this section, some Propositions on complete MPR are firstly introduced.

Note that a consistent MPR $A = (a_{ij})_{n\times n}$ can be precisely characterized by a weight vector $w = (w_1, w_2, \ldots, w_n)$ $(w_i > 0, \sum_{i=1}^{n} w_i = 1)$ such that $a_{ij} = \frac{w_i}{w_j}$, i.e., $\log a_{ij} = \log\frac{w_i}{w_j}$. Accordingly, a reasonable weight vector of an inconsistent MPR $A = (a_{ij})_{n\times n}$ is supposed to have this characterization as far as possible, i.e., the weight vector obtained should preserve the original information in $A = (a_{ij})_{n\times n}$ as much as possible. Then, given a complete MPR $A = (a_{ij})_{n\times n}$, its weight vector $w = (w_1, w_2, \ldots, w_n)$ can be obtained by the following optimization model:

$$\min D(A) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\log a_{ij} - \log\frac{w_i}{w_j}\right)^2 \tag{6}$$

$$\text{s.t. } w_i > 0, i \in I$$

$$\sum_{i=1}^{n} w_i = 1 .$$

Proposition 3.1. If MPR $A = (a_{ij})_{n\times n}$ is complete, then the optimal solution of (6) is

$$w_i = \frac{\prod_{j=1}^{n}\left(a_{ij}/a_{ji}\right)^{1/2n}}{\sum_{i=1}^{n}\prod_{j=1}^{n}\left(a_{ij}/a_{ji}\right)^{1/2n}}. \tag{7}$$

Proof. To determine the weight vector of $A = (a_{ij})_{n\times n}$, we construct the following function

$$f(w) = \sum_{i=1}^{n}\sum_{j=1}^{n}\left(\log a_{ij} - \log\frac{w_i}{w_j}\right)^2$$