Applied Soft Computing 43 (2016) 159–169

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

An approach to group decision making with hesitant information and its application in credit risk evaluation of enterprises

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ARTICLE INFO

Article history:
Received 25 August 2015
Received in revised form 5 February 2016
Accepted 5 February 2016
Available online 26 February 2016

Keywords:
Hesitant fuzzy element
Hesitant multiplicative preference relation
Group decision making
Error analysis
Credit risk evaluation

ABSTRACT

Hesitant multiplicative preference relation (HMPR) contains much more comprehensive information than the traditional multiplicative preference relations. The HMPR is a useful tool to help the decision makers express their preferences in group decision making under uncertainty. The key of group decision making with the HMPR is to derive the priority weights from the HMPR. Thus, an efficient and practical priority method should be put forward so as to ensure the reasonability of the final decision result. In order to do that, in this paper, we first introduce the expected value and the geometric average value of hesitant multiplicative element (HME) which is the component of the HMPR. Then from different perspectives, we utilize the error-analysis technique to put forward three novel methods for the priorities of the HMPR, i.e., the expectation value method, the geometric average value method, and the multiplicative deviation method. We also investigate the relationships among these methods, and develop an approach to group decision making with the HMPR by using the methods and the possibility degree formula. Finally, by constructing the indicator system for credit risk evaluation of supply chain enterprises, we make a detailed case study concerning Lu-Zhou-Lao-Jiao (the well-known liquor enterprise in China) to demonstrate our approach.

1. Introduction

Due to the limitation of the decision maker's cognition, the preference relations have been widely used to describe the decision maker's preferences in the decision making process. In 1977, Saaty introduced the multiplicative preference relations (MPRs), which are the basic components in the analytic hierarchy process (AHP) [1] and the analytic network process (ANP) [2]. Each element of a MPR is the preference value expressed by the decision maker over each pair of the alternatives according to the 1–9 scale [1]. Especially, if the decision maker prefers an alternative to another absolutely, then his/her preference can be expressed by the number 9; Conversely, it is expressed as the number 1/9.

In the complex decision making environment, when the decision makers are asked to provide their preferences by comparing pairs of alternatives, they may be uncertain and thus provide the insufficient preference information. Therefore, the traditional multiplicative preference relations have been extended into interval multiplicative preference relations [3], intuitionistic multiplicative preference relations [4], and linguistic multiplicative preference relations [5,6], which can help the decision makers express their uncertain preferences. For all kinds of multiplicative preference relations, the priority weight vectors are the final results, which can help the decision makers select the best alternative. Consequently, a lot of priority methods have been developed for MPRs, such as the eigenvector method [1], the goal programming methods [7,8], the least-squares method [9] and the logarithmic least-squares method [10], etc. In practical situations, the priority methods had better be simple and convenient so that the decision makers can get the decision results as quickly as they can. In such cases, Xu [11] introduced an error-analysis-based method for intuitionistic multiplicative preference relation, which needs less computation in actual applications.

However, in group decision making, the preferences of the decision makers are usually different or a decision maker cannot decide which one to choose among some preference values. Traditional multiplicative preference relation and its extensions express the decision group's preferences by presenting the preference

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http://dx.doi.org/10.1016/j.asoc.2016.02.010
1568-4946/

relation of each decision maker. It is difficult to analyze the decision group's preference information about each pair of alternatives. In order to integrate the group preference information, Xia and Xu [12] introduced the hesitant multiplicative preference relation (HMPR), in which the preference information provided by the decision makers through comparing a pair of alternatives is denoted by a hesitant multiplicative element (HME). For example, a group of decision makers discuss the degree to which an alternative is superior to another. Some of them provide 1/3, some provide 3, and the others provide 5. The decision makers' experiences and cognitions are different and none of them can be ignored, and thus, the preference information provided by the decision makers can be expressed as a HME {1/3, 3, 5}. In this situation, Zhu and Xu [13] extended the traditional analytic hierarchy process (AHP) to the hesitant AHP, and developed a hesitant multiplicative programming method (HMPM) as a new priority method to derive the ratio-scale priorities from HMPRs. Based on α-normalization and β-normalization, Zhang and Wu [23] developed a goal programming model and a convex combination method to derive the priority weights from HMPRs. Considering the twofold group decision making problem, Pérez-Fernández et al. [24] investigated the application of finite interval-valued hesitant fuzzy preference relations. Yu et al. [25] proposed two generalized hesitant fuzzy aggregation operators, and applied them to develop a procedure for group decision making.

However, the HMPRs contain all the information provided by the decision makers, and thus, these methods for group decision making based on the HMPRs are time-consuming and complex to obtain the priority weights. Moreover, because the information provided by the decision makers is important and none of the decision makers can be ignored, it is necessary to cover preference information as much as possible. The error-analysis technique can make the calculation process much easier and cover more information, which helps us obtain the decision results conveniently and precisely. The novelty of this paper is that it combines the error-analysis technique and the HMPR to support the group decision making process, which needs to cover all the hesitant information in a simple and convenient way. In this paper, we define the expected value and the geometric average value of the HME respectively. Then from different perspectives, we utilize the error-analysis technique to come up with three novel methods for the priorities of the HMPR, i.e., the expected value method, the geometric average value method, and the multiplicative deviation method, which can process hesitant information sufficiently. The methods have their own advantages and scopes of applications, so the decision makers can choose one of these methods according to different situations. Based on the proposed methods, we develop an error-analysis-based approach for group decision making.

The remainder of this paper is organized as follows: Section 2 introduces some basic concepts related to HME, HMPR and error analysis. In Section 3, we define the concepts of the expected value and the geometric average value of a HME respectively, and apply them as well as the error propagation formula to develop three priority methods for the HMPR. Then, we discuss the relationships among these methods, and develop an approach to group decision making with the HMPR by combining the possibility degree formula. In Section 4, we illustrate our approach with a credit risk evaluation problem of supply chain enterprises. The paper ends with some concluding remarks in Section 5.

2. Preliminaries

In what follows, we review some concepts and theorems related to the HMPR, which will be used in the next sections.

2.1. Hesitant fuzzy sets

In group decision making, the opinions of the decision makers are usually different from each other and none of them can be ignored. Thus, hesitant fuzzy sets have been introduced to deal with this issue. Torra [14] first defined the concept of hesitant fuzzy sets as follows:

Definition 1. [14]. Let X be a fixed set, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0, 1].

Based on the 1–9 scale [15], Zhu and Xu [13] introduced the hesitant multiplicative set (HMS):

Definition 2. [13]. Let X be a fixed set, a HMS is defined as:

$$Z = \{\langle x, z(x)\rangle | x \in X\} \tag{1}$$

where $z(x)$ is a subset of [1/9, 9] following the 1–9 ratio scale.

For convenience, $z = z(x)$ can be called a hesitant multiplicative element (HME). Since a HME may consist of several possible values, it can be considered as a hesitant judgment in the decision making environment. Based on the HMEs, in the following subsection, we will introduce the hesitant multiplicative preference relation (HMPR).

2.2. Hesitant multiplicative preference relation

Preference relations are a common tool to express the decision makers' preferences in the decision making problems. Among them, the multiplicative preference relation is the most widely used representation format, which was originally introduced by Saaty [1].

In order to represent the decision makers' preferences, the 1–9 scale [15] was constructed to describe the comparison relationships between two alternatives, as shown in Table 1.

Then, a multiplicative preference relation can be defined as follows:

Definition 3. [15]. A multiplicative preference relation H on the set X is defined as a reciprocal matrix $H = (h_{ij})_{n\times n} \in X \times X$ under the conditions:

$$h_{ij}h_{ji} = 1, \quad h_{ii} = 1, \quad h_{ij} > 0, \quad i, j = 1, 2, \ldots, n \tag{2}$$

where h_{ij} is interpreted as the ratio of the preference intensity of the alternative x_i to that of x_j.

In particular, $h_{ij} = 1$ implies indifference between the alternatives x_i and x_j; $h_{ij} > 1$ indicates that the alternative x_i is preferred to the alternative x_j, the greater h_{ij}, the stronger the preference intensity of the alternative x_i over x_j; $h_{ij} < 1$ means that the alternative x_j is preferred to the alternative x_i, the smaller h_{ij}, the greater the preference intensity of the alternative x_j over x_i.

To express the different preferences of the decision makers in the group decision making process, Xia and Xu [12] defined the hesitant multiplicative preference relation by combining the HFS and the MPR, shown as follows:

Definition 4. [12]. A hesitant multiplicative preference relation (HMPR) H on the set $A = \{A_1, A_2, \ldots, A_n\}$ is presented by a matrix $H = (h_{ij})_{n\times n} \subset A \times A$, where $h_{ij} = \{h_{ij}^t | t = 1, 2, \ldots, l_{h_{ij}}\}$ is a hesitant multiplicative element (HME) which indicates that all the possible degrees to which A_i is preferred to A_j. Moreover, h_{ij} should satisfy:

$$h_{ij}^t h_{ji}^{l_{h_{ji}}-t+1} = 1, \quad h_{ii} = \{1\}, \quad l_{h_{ij}} = l_{h_{ji}}, \quad i, j = 1, 2, \ldots, n \tag{3}$$

Xu [16] also defined the score of HME, which can be used to compare the relationship between two HMEs: