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Design of bio-inspired computational intelligence technique for solving steady thin film flow of Johnson–Segalman fluid on vertical cylinder for drainage problems

Muhammad Asif Zahoor Raja[a,*], Fiaz Hussain Shah[a], Abdul Ahad Khan[a], Najeeb Alam Khan[b]

[a] Department of Electrical Engineering, COMSATS Institute of Information Technology, Attock Campus, Attock 43600, Punjab, Pakistan
[b] Department of Mathematical Sciences, University of Karachi, Karachi 75270, Pakistan

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ABSTRACT

In the present study, bio-inspired computing technique is designed for solving governing mathematical relation for steady thin film flow of Johnson–Segalman fluid on vertical cylinder for drainage problems using Artificial Neural Networks (*ANNs*), genetic algorithms (*GAs*) and active-set algorithm (*ASA*). The strength of *ANN* modeling is exploited for the transformed equation of drainage problem which is derived from original partial differential equation using similarity transform. Training of design parameter of *ANNs* is carried out with evolutionary computing approach based on *GAs* hybrid with *ASA* for rapid local convergence. Design scheme is evaluated for number of cases of all four scenarios of drainage problem based on variations in Stokes number, Weissenberg number, ratio of viscosities, and slip parameters. Comparison of the results is made with Adams numerical method for each case in order to validate the accuracy of the proposed scheme. Results of statistical analysis in terms of performance measures based on mean, standard deviation, mean absolute deviation, root mean square error and Nash–Sutcliffe efficiency as well as their global variations further established the worth of the given scheme for each variant of drainage problem.

1. Introduction

The nonlinear differential equations are used to model number of applications arising in diverse field of engineering, science and technology. Particularly, the models of differential equation and their systems are incorporated broadly in chemical industry by the research community to study the dynamics in the field of Computational Fluid Mechanics (*CFM*) [1–4]. Normally, availability of exact solution is not possible for these governing mathematical models of fluids used in chemical industry due to their inherent complex nonlinear nature [5,6]. A sub-branch of *CFM* related with Computational Fluid Dynamics (*CFD*) provides a good alternate platform to many practical disciplines that incorporate empirical and semi-empirical laws based on flow measurement. Solving fluid dynamics problem, typically involves determination of various properties of the fluid, like velocity, pressure, density, and temperature, as functions of space and time, is a potential area of research. *CFD* problems have been used in wide range of applications, including obtaining the mass flow rate of crude oil [7,8], in petroleum products [9,10], for future prediction of weather [11,12], traffic engineering [13,14], *etc.* Theoretical researcher in mathematics has developed many numerical and analytical schemes with proven convergence to approximately solve these equations and few potential methods are Adomian Decomposition Method (*ADM*) [15,16], Finite Difference Schemes (*FDSs*) [17,18], Optimal Homotopy Perturbation Method (*OHPM*) [19,20], and Variational Iteration Method (*VIM*) [21,22]. All these recently introduced methodologies have their own limitations and gains in term of accuracy, convergence, robustness, and applicability, but based on well established deterministic procedures. Whereas numerical solvers based on stochastic procedures are relatively less exploited to solve nonlinear problems of *CFD*.

Stochastic techniques based on computational intelligence methods using feed-forward Artificial Neural Networks (*ANNs*) are considered to be fundamental in machine learning and pattern recognition. Well-established strength of these *ANNs* solvers have been widely used to solve the linear and nonlinear differential equations as well. However, these procedures are not exploited aggressively to solve potential *CFD* problems due to non-availability of expensive computational platform. Recently, soft computing or machine learning or computational intelligence techniques based on *ANNs* optimized with swarming and evolutionary techniques have been utilized to solve number of problems [23–25]. For instant, few applications of these solvers to study the dynamics of nonlinear Van der Pol oscillators

* Corresponding author. Tel.: +92 3009893800.
E-mail addresses: muhammad.asif@ciit-attock.edu.pk, rasifzahoor@yahoo.com, asif.phdee10@iiu.edu.pk (M.A.Z. Raja), fsgillani@gmail.com (F.H. Shah), ahadkhanyousafzai@hotmail.com (A.A. Khan), njbalam@yahoo.com (N.A. Khan).

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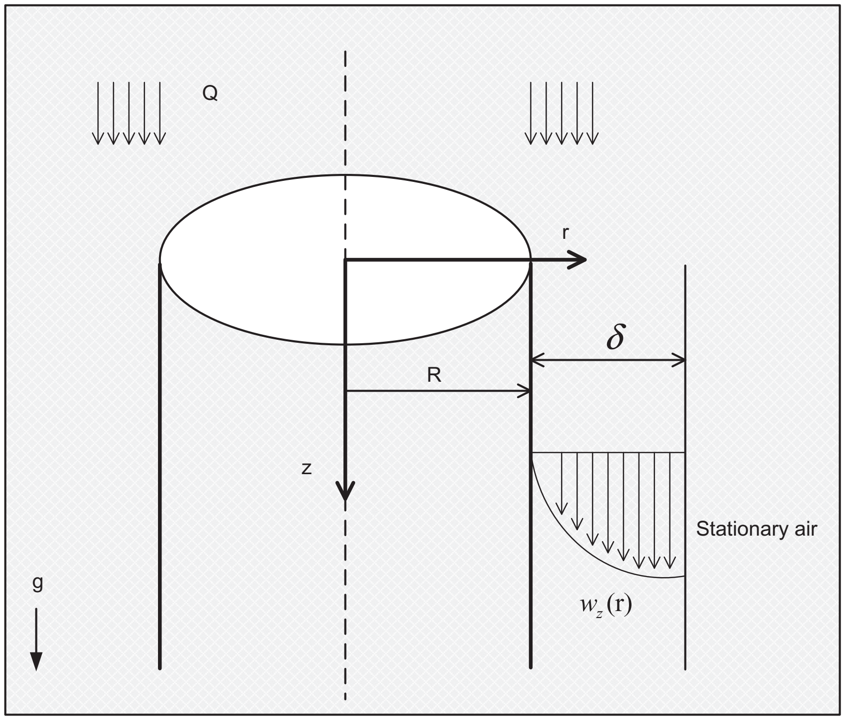

Fig. 1. Schematic diagram of drainage problem.

with both stiff and non-stiff cases, nonlinear singular problems represented with Emden-Fowler type equations, fluid dynamics problems in the presence of high magnetic field based on Jeffery-Hamel flow equations, nonlinear first Painlevé equation, nonlinear Troesch's problem arising in plasma physics and two dimensional nonlinear Bratu-type equations in transformed structure and boundary values problems arising in fuel ignition model of combustion theory ([26,27] and the reference therein). These stochastic algorithms have much strengthen their intrinsic worth with applicability to solve non-linear fractional differential equations, particularly, well-known fractional order systems governed with Riccati and Bagley–Torvik equations [28,29]. These are the motivation factor for authors for exploration and exploitation in this domain and provide an alternate, accurate, reliable and robust platform for solving drainage problem of fluid dynamics.

The aim of this study is to find approximate solutions of fluid dynamics model based on drainage problem for thin film flow of non-Newtonian fluid with the help of artificial intelligence technique, and compare the performance with existing analytical and numerical solvers. The study of velocity profile of the fluid in drainage problem for different scenarios based on values of Stokes number, Weissenberg number, ratio of viscosities and slip parameters is made. Comparative studies are carried out with the state of art numerical scheme based on Adams method for single as well as multiple independent runs for each case of drainage problem.

2. System model: drainage problem

In this section, system model for steady thin film flow of non-Newtonian Johnson–Segalman fluid on vertical cylinder for drainage problems is briefly described [30].

The flow of an incompressible fluid by neglecting the thermal effect is given mathematically by the field equation as:

$$div\,\mathbf{V} = 0,$$
$$\rho\frac{D\mathbf{V}}{Dt} = div\,\boldsymbol{\sigma} + \rho\mathbf{f}, \tag{1}$$

here $\mathbf{V}$ representing the velocity vector of the fluid, $\boldsymbol{f}$ stands for the body force per unit mass, ρ is the constant density, $\frac{D}{Dt}$ is the material time derivative and σ is the Cauchy stress-tensor (*CST*). The *CST* in case of Johnson–Segalman fluid (*JSF*) is related with fluid motion through the following expressions [30,31]:

$$\boldsymbol{\sigma} = -p\mathbf{I} + \mathbf{T}, \tag{2}$$

$$\mathbf{T} = 2\mu\mathbf{D} + \mathbf{S}, \tag{3}$$

$$\mathbf{S} + m\left[\frac{D\mathbf{S}}{Dt} + \mathbf{S}(\mathbf{W} - a\mathbf{D}) + (\mathbf{W} - a\mathbf{D})^T\mathbf{S}\right] = 2\eta\mathbf{D}, \tag{4}$$

$$\frac{D\mathbf{S}}{Dt} = \frac{\partial\mathbf{S}}{\partial t} + [grad\,\mathbf{S}]V, \tag{5}$$

where $\mathbf{D}$ and $\mathbf{W}$ are the symmetric and skew symmetric part of the velocity gradient, respectively, which are given below for $\mathbf{L} = grad\,\mathbf{V}$,

$$\mathbf{D} = \frac{1}{2}[\mathbf{L} + \mathbf{L}^T], \quad \mathbf{W} = \frac{1}{2}[\mathbf{L} - \mathbf{L}^T], \tag{6}$$

while $-p\mathbf{I}$ is a part of the stress due to incompressibility constraints, μ and η are representing the viscosities, m denotes the relaxation time and a is the slip parameter. *JSF* model transforms to the Maxwell and Newtonian fluid model in case of $a = 1$, $\eta = 0$, and $\eta = m = 0$, respectively.

2.1. Problem formulation

Geometry of the problems is shown in Fig. 1, in which a non-Newtonian Johnson–Segalman fluid is considered to be falling on the outer surface of vertical cylinder of radius R with infinitely long length, in the form of a thin, uniform axisymmetric film with thickness δ, in contact with stationary air [30].

With the assumption that the flow is steady, the surface tension is zero, and the pressure is atmospheric pressure; the velocity field is given as:

$$\mathbf{V} = (0,\ 0,\ w(r)). \tag{7}$$

By considering the radial direction normal to the cylinder and the z-direction along the cylinder in downward direction as shown in

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