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A rough set method for the minimum vertex cover problem of graphs

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ABSTRACT

The minimum vertex cover problem is a classical combinatorial optimization problem. This paper studies this problem based on rough sets. We show that finding the minimal vertex cover of a graph can be translated into finding the attribute reduction of a decision information table. At the same time, finding a minimum vertex cover of graphs is equivalent to finding an optimal reduct of a decision information table. As an application of the theoretical framework, a new algorithm for the minimum vertex cover problem based on rough sets is constructed. Experiments show that the proposed algorithm gets better performance in terms of the ratio value when compared with some other algorithms.

1. Introduction

Q4 Graph theory is a useful tool for data analysis and knowledge representation in computer science. The minimum vertex cover problem (MVCP) is a classical graph optimization problem, which is to find a minimal vertex cover with the least number of vertices [6]. Except the application in graph theory, MVCP also has been used in a wide variety of real-world applications, such as crew scheduling [42], VLSI design [3,23], nurse rostering [7] and industrial machine assignments [55].

As shown in [5,15,32], the minimum vertex cover computation can be translated into the calculation of prime implicants of a Boolean function. Although one can generate all the minimal vertex covers (or a minimum vertex cover) of a graph by using the Boolean operation, it is a well known NP-hard optimization problem [15,27]. There are a number of approximation algorithms that have been proposed for this problem in the literature [1,2,14,18,20–22,35]. In [19], Gomes et al. conducted a comparative study of three approximation algorithms for MVCP via some numerical experiments. The results showed that the Greedy algorithm was faster than both the Round and Dual-LP algorithms, and it also had a superior performance on the ratio value. Avis and Imamura proposed a simple and effective approximation algorithm called the list heuristics for MVCP [1]. In recently, a competent algorithm called Vertex Support Algorithm (VSA) was proposed for efficiently solving MVCP [2].

Rough set theory is another effective tool for data analysis and knowledge discovery. The notion of attribute reduction plays an important role in the theory of rough sets. An attribute reduct is a minimal subset of attributes that provides the same classification ability as the whole set of attributes [36,37]. So far, it has been widely used in pattern recognition [25,26,45], knowledge discovery [65] and machine learning [31].

Many approaches have been proposed for the attribute reduction [9,10,30,33,56,57,59]. A beautiful theoretical result is based on the notion of a discernibility matrix. Skowron and Rauszer [43] showed that the set of all reducts is in fact the set of prime implicants of the discernibility function. However, as was shown by Wong [54], finding the set of all attribute reducts or an optimal reduct (a reduct with the minimum number of attributes), is an NP-hard problem. Various heuristic methods for the attribute reduction such as positive-region methods [24,39], information entropy methods [29,38,44,51,58,63] and discernibility matrix methods [8,40,49,50] have been developed.

As we have discussed above, attribute redacts and minimal vertex covers can be obtained via the Boolean logical operation. It seems that there is some kind of natural connection between the two problems. The purpose of this paper is mainly to study MVCP based on rough sets. In fact, Wang et al. [52] studied MVCP from a viewpoint of covering-based rough sets. However, they did not propose any efficient algorithm for MVCP. Kulaga et al. investigated the attribute reduction of a consistent decision table based on graph theory [28]. The framework proposed in this paper is quite different

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from that of [28,52]. We show that the problem of finding the minimal vertex cover of a graph is equivalent to the problem of finding the attribute reduction of a decision information table. Also, finding a minimum vertex cover of graphs is equivalent to finding an optimal reduct of its decision information table. What's more, we present an effective algorithm based on rough sets for MVCP. This study may open new research directions and provide new methods for MVCP.

The remainder of this paper is organized as follows. In Section 2, some basic notions related to rough sets and graph theory are introduced. In Section 3, a new decision information table induced from a given graph is constructed, and the relationship between the attribute reduction of the derivative decision information table and the minimal vertex cover of the graph is studied. In Section 4, a new approximate algorithm for MVCP based on rough sets is presented. Some numerical experiments are given to show the effectiveness of the proposed method in Section 5. Finally, some conclusions are drawn in Section 6.

2. Preliminaries

In this section, we briefly introduce some basic notions and results about rough sets and graph theory [6,37,46].

2.1. Vertex covers in graph theory

A graph is a pair $G=(V,E)$ consisting of a set V of vertices and a set E of edges such that $E\subseteq V\times V$. Two vertices are adjacent if there is an edge joining them, and the vertices are then incident with such an edge. Two or more edges that link the same pair of vertices are said to be parallel edges. An isolated vertex is a vertex not adjacent to any other vertex. A loop is an edge with the same ends. The edge of a graph may be directed (asymmetric) or undirected (symmetric). An undirected graph is one in which the edges are symmetric.

A vertex cover of a graph G is a subset $K\subseteq V$ such that every edge of G has at least one end in K. A vertex cover is minimal if none of its proper subsets is itself a vertex cover. A minimum vertex cover is a vertex cover with the least number of vertices. Note that a minimum vertex cover is always minimal but not necessarily vice versa. Observe that a minimal vertex cover is not necessarily unique, it is also true for the minimum vertex cover. All the minimal vertex covers of a graph can be obtained via Boolean formulaes.

Given a graph $G=(V,E)$ and $e\in E$, let $N(e)$ denote a set of vertices connected by the edge e. Denote $\mathcal{N}=\{N(e)|e\in E\}$. Now we define a function f_G for G as follows, which is a Boolean function of m Boolean variables $v_1^*, v_2^*, \cdots, v_m^*$ corresponding to the vertices $v_1, v_2, \cdots, v_m$, respectively.

$$f_G(v_1^*, v_2^*, \cdots, v_m^*)=\wedge\{\vee N(e)|N(e)\in\mathcal{N}\},$$

where $\vee N(e)$ is the disjunction of all variables v^* such that $v\in N(e)$.

The following lemma gives a method for computing the minimal vertex covers of a given graph.

Lemma 1 (([15,32]).). *Let $G=(V,E)$ be a graph. A vertex subset $K\subseteq V$ is a minimal vertex cover of G iff $\bigwedge_{v_i\in K}v_i^*$ is a prime implicant of the Boolean function f_G.*

Lemma 1 shows that if

$$f_G(v_1^*, v_2^*, \cdots, v_m^*)=\wedge\{\vee N(e)|N(e)\in\mathcal{N}\}=\bigvee_{i=1}^{t}(\bigwedge_{j=1}^{s_i}v_j^*),$$

where $\bigwedge_{j=1}^{s_i}v_j^*, i\le t$, are all the prime implicants of the Boolean function f_G, then $K_i=\{v_j|j\le s_i\}, i\le t$, are all the minimal vertex covers of G. The set of all minimal vertex covers of a graph G is denoted by $\mathcal{C}(G)$. We will also write v_i instead of v_i^* in the discussion to follow.

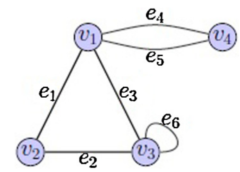

Fig. 1. The graph of Example 1.

Example 1. Let $G=(V, E)$ be the following graph with $V=\{v_1, v_2, v_3, v_4\}$ and $E=\{e_1, e_2, e_3, e_4, e_5, e_6\}$ (Fig. 1).

We have the Boolean function:

$$f_G(v_1, v_2, v_3, v_4)=(v_1\vee v_2)\wedge(v_2\vee v_3)\wedge(v_1\vee v_3)\wedge(v_1\vee v_4)\wedge(v_1\vee v_4)\wedge v_3.$$

After simplification, we have f_G in prime implicants as:

$$f_G(v_1, v_2, v_3, v_4)=(v_1\wedge v_3)\vee(v_2\wedge v_3\wedge v_4).$$

Hence G has two minimal vertex covers: $K_1=\{v_1, v_3\}$ and $K_2=\{v_2, v_3, v_4\}$. K_1 is the unique minimum vertex cover.

2.2. Attribute reduction with rough sets

An information table can be seen as a pair $S=(U, A)$, where U and A, are finite, non-empty sets called the universe (a set of objects) and the set of attributes, respectively. With each attribute $a\in A$, we define an information function $a: U\longrightarrow V_a$, where V_a is the set of values of a, called the domain of a.

Each non-empty subset $B\subseteq A$ determines an indiscernibility relation:

$$R_B=\{(x, y)\in U\times U|a(x)=a(y), \forall a\in B\}.$$

Obviously, R_B is an equivalence relation on U, it forms a partition $U/B=\{[x]_B|x\in U\}$, where $[x]_B$ denotes the equivalence class containing x w.r.t. B, i.e., $[x]_B=\{y\in U|(x, y)\in R_B\}$.

Let $B\subseteq A$ and $X\subseteq U$, the two sets

$$\underline{B}X=\{x\in U|[x]_B\subseteq X\}, \overline{B}X=\{x\in U|[x]_B\cap X\neq\emptyset\},$$

are called the lower and the upper approximation of X w.r.t. B, respectively. The lower approximation $\underline{B}X$ is also called the positive region of X.

A decision table is a special information table with the form $S=(U, A\cup\{d\})$, where (U, A) is an information table and $d\notin A$. Usually, A is called the conditional attribute set and d is the decision attribute. Suppose $U/d=\{D_1, D_2, \cdots, D_r\}$ are the equivalence classes induced by d. The positive region of d w.r.t. B, denoted by $POS_B(d)$, is defined as $POS_B(d)=\bigcup_{i=1}^{r}\underline{B}D_i$.

Given a decision table $S=(U, A\cup\{d\})$, an attribute subset $B\subseteq A$ is a reduct (also called a relative reduct) of S if B is a minimal set such that $POS_B(d)=POS_A(d)$. Various approaches to attribute reduction have been proposed in the literature. For our purpose, we introduce the following method based on the discernibility matrix and logical operation [43]. By the discernibility matrix method, one can get all the reducts of a decision table.

Let $S=(U, A\cup\{d\})$ be a decision table with n objects and $(x, y)\in U\times U$. We define

$$M(x, y)=\begin{cases}\{a\in A|a(x)\neq a(y)\}, & (x, y)\in DIS,\\ \emptyset, & otherwise,\end{cases}$$

where DIS is the set consisting of $(x, y)\in U\times U$ satisfying one of the following conditions: (1) $x\in POS_A(d)$ and $y\notin POS_A(d)$; (2) $x\notin POS_A(d)$ and $y\in POS_A(d)$; (3) $x, y\in POS_A(d)$ and $d(x)\neq d(y)$. $M(x, y)$ is referred to as the discernibility attribute set of x and y in S, and $\mathcal{M}=\{M(x, y)|(x, y)\in U\times U\}$ is called the discernibility set of

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