Applied Soft Computing 40 (2016) 64–69

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

On solving a multiobjective fixed charge problem with imprecise fractional objectives

M. Upmanyu [a,*], R.R. Saxena [b]

[a] Department of Mathematics, University of Delhi, Delhi, India
[b] Department of Mathematics, Deen Dayal Upadhyaya College, University of Delhi, Delhi, India

ARTICLE INFO

Article history:
Received 10 May 2014
Received in revised form 11 August 2015
Accepted 3 October 2015
Available online 22 November 2015

Keywords:
Fixed charge problem
Multiobjective programming
Fractional programming
Fuzzy objective function

ABSTRACT

The fixed charge problem is a special type of nonlinear programming problem which forms the basis of many industry problems wherein a charge is associated with performing an activity. In real world situations, the information provided by the decision maker regarding the coefficients of the objective functions may not be of a precise nature. This paper aims to describe a solution algorithm for solving such a fixed charge problem having multiple fractional objective functions which are all of a fuzzy nature. The enumerative technique developed not only finds the set of efficient solutions but also a corresponding fuzzy solution, enabling the decision maker to operate in the range obtained. A real life numerical example in the context of the ship routing problem is presented to illustrate the proposed method.

1. Introduction

The fixed charge problem is one of the interesting applications of the mixed integer programming problem having a practical use both in business and industry and was initialized by Hirsch and Dantzig in 1968 [13]. In this nonlinear programming problem, there is a cost associated with performing an activity at a nonzero level which does not depend on the level of the activity. The existence of these fixed charges associated with the activities in the objective function produce a nonlinear programming problem. The fixed charges may be expressed as a measure other than cost, such as the setup time of getting a machine into service. Up to now, it has been widely applied in many decision-making and optimization problems like the warehouse or plant location decisions, wherein there is a charge associated with opening the facility; in transportation problems where there are fixed charges for transporting goods between supply points and demand points.

Only a few exact methods for solving it are present in literature. Based on the branch and bound method, Steinberg [20] provided an exact method for small problems. On the other hand, various approximate solution methods have been developed by Cooper [7], Murty [18], Cooper and Drebes [8], Walker [21] based on adjacent extreme point algorithms. In more recent times, the emphasis has shifted to the study of the fixed cost transportation problem which is mainly solved by ranking of the extreme points and the branch and bound method. This is because the problem reduces to that of minimising a concave function over a bounded convex set. The resulting optimum is taken at one or more of the extreme points of the feasible region; but for a non-degenerate problem with positive fixed costs, every extreme point of the feasible region is a local minimum. In [1], Adlakha and Kowalski use the Balinski approximation method introduced for the fixed charge transportation problem and apply the same for solving the fixed charge problem.

A natural generalization of the linear objective function is a problem having a linear fractional (or hyperbolic) objective function, giving rise to the fractional programming problem which involves the optimization of one or several ratios of functions subject to some constraints [5,19]. Such objectives arise in economical models when the goal is to optimize profit per unit investment type functions. In some applications where the efficiency of a system is to be optimized, the efficiency may be characterized by a ratio of technical and/or economical terms which leads to a fractional program. In 1971, Almogy and Levin [2] introduced the fractional fixed charge problem which arises in numerous applications, where the measure of economic performance is the time rate of earnings or profit (equivalent to an interest rate on capital investment). They discussed its application to the ship routing problem by optimizing the profit per unit time over a given route. The criteria then for choosing a route is that the resulting profit per unit time exceeds the fixed cost per unit time at the port from where all routes originate and terminate. Recent applications of the fixed charge problem having the dual concept of fixed and variable cost have been seen in the network design flow problems [12] and the facility location problems [10]. Also, Arora [3] developed a systematic extreme point enumeration technique, which provides an exact solution to the fixed charge problem.

In the fixed charge problem though, the parameters in the objective are supplied according to the decision makers' requirements, who in most cases is unable to provide this precise information. In such a case, we formulate them as fuzzy numbers, in other words the objective function can be fuzzified and a leverage is provided to the decision maker to operate. To the authors' knowledge, no work has yet been done in developing a solution procedure for solving a multiobjective fixed charge problem with linear fractional objective functions having fuzzy parameters. In this paper we intend to provide the required algorithm and a fuzzy solution for the same. Preliminary concepts of fuzzy numbers are studied in Section 2. The problem under discussion is formulated, and related concepts of Pareto efficiency are defined in Section 3. The algorithm developed in Section 4 is supported by a real life numerical example in the context of the ship routing problem in Section 5.

2. Fuzzy numbers

In this section we review some fundamentals of fuzzy numbers and ranking functions which will be used through the remainder

* Corresponding author. Tel.: +91 7710024909.
E-mail addresses: muditaupmanyu@gmail.com (M. Upmanyu), ratnesh65@gmail.com (R.R. Saxena).

http://dx.doi.org/10.1016/j.asoc.2015.10.008
1568-4946/

of this paper. We shall use the most common definition of a fuzzy number given by Dubois and Prade [9].

Fuzzy number. *A fuzzy number* $\tilde{A}$ *is a convex normalized fuzzy set of the real line* $\mathbb{R}$ *such that:*

(i) $\exists$ *unique* $x_0 \in \mathbb{R}$ *such that* $\mu_{\tilde{A}}(x_0) = 1$, *and*
(ii) $\mu_{\tilde{A}}$ *is piecewise continuous.*

Triangular fuzzy number. *A fuzzy number* $\tilde{A}$ *is a triangular fuzzy number if its membership function* $\mu_{\tilde{A}}$ *is of the form*

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{x-c}{b-c}, & b \le x \le c \\ 0, & \text{otherwise} \end{cases}$$

where a, b and c are real numbers. It is denoted by $\tilde{A} = (a, b, c)$.

Let us denote the set of fuzzy numbers by $F(\mathbb{R})$. The basic arithmetic operations between two triangular fuzzy numbers are given below [15].

Let $\tilde{A} = (a, b, c), \tilde{B} = (x, y, z) \in F(\mathbb{R})$, then

(i) *Addition:* $(\tilde{A} \oplus \tilde{B}) = (a + x, b + y, c + z)$.
(ii) *Scalar multiplication:*

$$(k \otimes \tilde{A}) = \begin{cases} (ka, kb, kc) & \text{if } k > 0 \\ (kc, kb, ka) & \text{if } k < 0 \end{cases}$$

(iii) *Multiplication:* $(\tilde{A} \otimes \tilde{B}) = (\min\{ax, az, cx, cz\}, by, \max\{ax, az, cx, cz\})$
(iv) *Division:* $(\tilde{A} ø \tilde{B}) = (\min\{a/x, a/z, c/x, c/z\}, b/y, \max\{a/x, a/z, c/x, c/z\})$.
(v) *Non-negativity:* $\tilde{A}$ is said to be a non-negative triangular fuzzy number iff $a \ge 0$.

Since two fuzzy numbers cannot be compared due to the absence of any linear ordering in the set $F(\mathbb{R})$, a simple concept of ranking functions is employed for such a comparison. A ranking function is a mapping $R : F(\mathbb{R}) \to \mathbb{R}$ which maps each fuzzy number into a point on the real line where a natural order exists. Baas and Kwakernaak [4] are among the pioneers in this area. Several methods of ranking fuzzy subsets have been proposed and comparisons of these methods have been reported in [6,11]. In this paper we consider the ranking of triangular fuzzy numbers using the concept of total integral value given by Liou and Wang [17]. The left integral value is used to reflect the pessimistic viewpoint and the right integral value is used to reflect the optimistic viewpoint of the decision maker. A convex combination of the left and right integral values through an index of optimism [16] is called the total integral value. Here for a given level of priority $\alpha \in [0, 1]$, the total integral value of the triangular fuzzy number $\tilde{A} = (a, b, c)$ is taken to be the ranking function defined as

$$R(\tilde{A}) = \frac{1}{2}[\alpha c + b + (1 - \alpha)a]$$

We can thus define orders on $F(\mathbb{R})$ as follows. Let $\tilde{A}, \tilde{B} \in F(\mathbb{R})$. Then,

$\tilde{A} \succcurlyeq \tilde{B}$ if and only if $R(\tilde{A}) \ge R(\tilde{B})$,
$\tilde{A} \succ \tilde{B}$ if and only if $R(\tilde{A}) > R(\tilde{B})$,
$\tilde{A} \approx \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$.

Also we write $\tilde{A} \preccurlyeq \tilde{B}$ iff $\tilde{B} \succcurlyeq \tilde{A}$ and $\tilde{A} \prec \tilde{B}$ iff $\tilde{B} \succ \tilde{A}$.

3. Problem formulation

Many problems of economics and logistics involve the planning of a large number of interdependent activities in as economical a way as possible. There exists an important class of nonlinear programming problems of the form

$$\text{(FCP)} \quad \text{Minimize} \quad \Phi(X) = \sum_{j=1}^{n} \alpha_j x_j + \beta_j \delta(x_j)$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad i = 1, 2, \ldots, m$$
$$x_j \ge 0, \qquad j = 1, 2, \ldots, n$$

where $\alpha_j, \beta_j, j = \{1, 2, \ldots, n\}$ are constants, and

$$\delta(u) = \begin{cases} 0, & \text{if } u = 0 \\ 1, & \text{if } u > 0 \end{cases}$$

This problem with a nonlinear objective function and linear constraints is called the Fixed Charge Problem (FCP) and the constants β_j are the fixed charges associated with the jth activity. These fixed charges are incurred if a new activity is engaged in at a positive level, which leads to a new charge to be borne by the firm.

The fractional fixed charge problem (FFCP) discussed by Almogy and Levin [2] is described as follows

$$\text{(FFCP)} \quad \text{Minimize} \quad Z(X) = \frac{\sum_{j=1}^{n} c_j x_j + \sum_{j=1}^{n} F_j}{\sum_{j=1}^{n} d_j x_j + \sum_{j=1}^{n} f_j + \gamma}$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad i \in I = 1, 2, \ldots, m \tag{1}$$

$$x_j \ge 0, \quad j \in J = 1, 2, \ldots, n \tag{2}$$

where γ is a constant. It is assumed that $\sum_{j=1}^{n} d_j x_j + \sum_{j=1}^{n} f_j + \gamma > 0$ over the solution set. The numbers F_j and f_j, $j \in J$ are the fixed charges associated with the decision variable x_j and are formulated in the following manner. If the quantity of the variable x_j is less than a value A_j say, then the fixed charge is k_{j_1} units. When the quantity exceeds or is equal to A_j, an additional fixed charge k_{j_2} is incurred. Then,

$$F_j = \text{total fixed charge associated with the variable } x_j$$
$$= \delta_1 k_{j_1} + \delta_2 k_{j_2}$$

$$\text{where} \quad \delta_1 = \begin{cases} 1, & \text{if } 0 < x_j < A_j \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad \delta_2 = \begin{cases} 1, & \text{if } x_j \ge A_j \\ 0, & \text{otherwise} \end{cases}$$

Note here that F_j is a step function and in this example it has two steps. In [2] it is established that the optimal solution of the fractional fixed charge problem occurs at an extreme point of the feasible region defined by (1) and (2).

The mathematical model of the problem under consideration which is the multiobjective fractional fixed charge problem (FMCP) with fuzzy coefficients is

$$\text{(FMCP)} \quad \text{Minimize} \quad \tilde{Z}(X) = (\tilde{Z}_1(X), \tilde{Z}_2(X), \ldots, \tilde{Z}_p(X))$$

$$\text{subject to} \quad \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad i \in I = \{1, 2, \ldots, m\}$$
$$x_j \ge 0, \qquad j \in J = \{1, 2, \ldots, n\}$$