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A novel risk attitudinal ranking method for intuitionistic fuzzy values and application to MADM

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ABSTRACT

The ranking of intuitionistic fuzzy sets (IFSs) is very important for the intuitionistic fuzzy decision making. The aim of this paper is to propose a new risk attitudinal ranking method of IFSs and apply to multi-attribute decision making (MADM) with incomplete weight information. Motivated by technique for order preference by similarity to ideal solution (TOPSIS), we utilize the closeness degree to characterize the amount of information according to the geometrical representation of an IFS. The area of triangle is calculated to measure the reliability of information. It is proved that the closeness degree and the triangle area just form an interval. Thereby, a new lexicographical method is proposed based on the intervals for ranking the intuitionistic fuzzy values (IFVs). Furthermore, considered the risk attitude of decision maker sufficiently, a novel risk attitudinal ranking measure is developed to rank the IFVs on the basis of the continuous ordered weighted average (C-OWA) operator and this interval. Through maximizing the closeness degrees of alternatives, we construct a multi-objective fractional programming model which is transformed into a linear programme. Thus, the attribute weights are derived objectively by solving this linear programme. Then, a new method is put forward for MADM with IFVs and incomplete weight information. Finally, an example analysis of a teacher selection is given to verify the effectiveness and practicability of the proposed method.

1. Introduction

Fuzzy sets (FSs) proposed by Zadeh [1] can be used to express the uncertain information. Due to the influence of subjective factors, the decision maker (DM) sometimes relies on intuition and experience to evaluate the information in real-life decision problems. Thus, it is often that DM exhibits some hesitation degrees for the assessments. Atanassov [2] proposed the intuitionistic fuzzy sets (IFSs) which assign the membership degree, non-membership degree and the degree of hesitation to each element. It is just a suitable tool to cope with the vagueness and hesitancy originating from imprecise knowledge or information. Since IFS simultaneously contains membership and non-membership degrees, it is more flexible and practical than FSs in dealing with ambiguity and uncertainty [3–10].

IFS has been widely applied in the multi-attribute decision making (MADM) and multi-attribute group decision making (MAGDM) [11–13]. In these applications of IFSs, how to give the order relationship of intuitionistic fuzzy values (IFVs) is a critical issue. In 1999, Atanassov [14] proposed an Atanassov's order of IFVs. Atanassov's order can be seen as a natural order, namely, it is the foundation of other ranking methods. Subsequently, lots of works have done in the topics of ranking IFVs and measuring the entropy or amount of knowledge conveyed by IFVs in recent years. The existing achievements can be roughly divided into two classes.

The first class is to use the combinatorial calculations on the two or three functions (membership function, non-membership function, and hesitation margin) [15–19]. Chen and Tan [15] paid attention to the differences between the membership function and non-membership function and then presented a score function to evaluate the degree of score of IFVs. Wu and Chiclana [16] defined a new score function and its value is between 0 and 1. The new score function and the score function in [15] are ordering mathematically equivalent in that they will lead to the same ordering of

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alternatives when the using the same ordering rule. However, the score function cannot distinguish many IFVs, Hong and Choi [17] proposed an accuracy function to evaluate the degree of accuracy of IFVs by summing the membership function and non-membership function. Xu [18] gave an algorithm to rank IFVs by combining the score function and accuracy function. Then Liu and Wang [19] suggested a new score function by means of the intuitionistic fuzzy point operator. However, neither there had basis for this allegation, nor they gave an in-depth analysis of it.

The second class is to use the geometrical representation [20–29]. It has been proved that the representation is effective to bring about intuitively appealing results in solving many intuitionistic fuzzy problems, such as distance [20–24] and entropy [20,22,24–26,28,29]. Szmidt and Kacprzyk [20] introduced a measure for ranking IFVs by considering the amount and reliability of information related to an IFS. Guo and Li [21] applied the ranking measure in [20] to an attitudinal-based intuitionistic fuzzy decision model. Guo [22] pointed out that Szmidt and Kacprzyk's order [20] still lead to undesirable results to some extent. Then Guo [22] developed a new technique for ranking IFVs based on amount of information, and then extended to the attitudinal-based technique which took the DM's attitude into account. Ouyang and Pedrycz [23] combined Atanassov's order [14] and Szmidt and Kacprzyk's order [20] to propose a new admissible order for IFVs, which is proved to a lexicographical one. Chen et al. [24] proposed a score function for IFVs, which considers the distance from the positive and negative ideal IFVs and DM's attitude. Szmidt et al. [25] concerned the intrinsic relationship between the positive and negative information and the hesitation margin to express the amount of knowledge conveyed by IFVs. Pal et al. [26] presented new axiomatic characterizations of the non-probabilistic entropy measures associated with an IFS. Szmidt and Kacprzyk [27] investigated a non-probabilistic-type entropy measure for IFSs, which is a result of a geometric interpretation of IFSs and uses a ratio of distances. Szmidt and Kacprzyk [28] continued the previous paper to study the entropy of IFVs by examining their differences. Szmidt and Kacprzyk [29] generalized the entropy of IFSs by considering all three functions. One important case occurring in the above is that the hesitation margin, as one of the three functions, is a key role in determining the amount of information [21–29].

The aforementioned methods seem to be effective to rank IFVs. However, they have some disadvantages as follows:

(1) The methods [15–19] utilized the traditional tools involving the membership function and non-membership function, which sometimes may suffer from counterintuitive ranking results or the unobtainable results.
(2) Although the papers [20–23,25,27,29] used the distance to define the measures for ranking the IFVs, they only considered the distance with the positive ideal point $M(1, 0, 0)$ and neglected the negative ideal point $N(0, 1, 0)$.
(3) The methods [25–29] measured the uncertainty and the amount of information with entropy which cannot distinguish one IFV x and its corresponding complement x^c. For example, for a given IFV x, the measure $K(x)$ (see Eq. (9) in Section 2.2) is equal to $K(x^c)$, that is to say, x and x^c have the identical uncertainty. Hence, such a measure cannot be used to rank the IFVs.

Especially, in real-life decision problems, different DMs have diverse attitudes towards risk. It is necessary and natural to incorporate DM' risk preference into the ranking method, which is useful and flexible in real-world applications. Yager [30] introduced the continuous ordered weighted average (C-OWA) operator in which the weights can imply DM's attitude. Then the C-OWA operator is applied to some ranking methods. Guo [22] discussed the role that DM's attitude can play in decision making under uncertainty and proposed an attitudinal-based ranking technique. Wu and Chiclana [31] defined an attitudinal expected score function for interval-valued IFVs. Jin et al. [32] developed an interval-valued intuitionistic fuzzy continuous weighted entropy based on the C-OWA operator.

Therefore, to overcome these disadvantages, new measurements of information amount and reliability of IFVs are devised and a new lexicographic method is proposed to rank IFVs in this paper. Then we further develop a risk attitudinal measure for ranking IFVs considering the DM's risk attitude. Finally, a novel method is proposed for MADM with IFVs and incomplete weight information. The main contributions of this work are summarized as three aspects:

(1) New measurements of information amount and reliability of IFVs are designed from the angle of geometric meaning. These new measurements consider the positive and negative ideal points and the area of triangle jointly. It is proved that the closeness degree and the area just form an interval. Thereby, a new lexicographic method is developed to rank IFVs, which is very simple and effective.
(2) Considering that the closeness degree and the area form an interval, we further define a novel risk attitudinal measure for ranking IFVs based on the C-OWA operator. Thus, this risk attitudinal measure is more theoretically reasonable and convincing than that defined in Guo [22].
(3) A new approach to determining the attribute weights objectively is presented through constructing multi-objective fractional programming model. It is dexterously converted into linear programming to resolve by the Charnes and Cooper transformation.

The remainder of this paper unfolds as follows. In Section 2, some concepts of IFSs are recalled and some typical ranking methods are reviewed. Section 3 defines new measurements of information amount and reliability of IFVs according to geometrical representation and then proposes a new lexicographic method for ranking IFVs. Furthermore, a new risk attitudinal measure is defined to ranking IFVs in Section 4. In Section 5, a new method is proposed for MADM with IFVs and incomplete weight information. A teacher selection example and the comparison analyses are given in Section 6. Finally, concluding remarks are made in Section 7.

2. Intuitionistic fuzzy sets

In this section, we recall some basic concepts of IFSs, including the definition, operation laws and Hamming distances. In the meantime, some existing ranking methods of IFVs are reviewed.

2.1. Preliminaries of intuitionistic fuzzy sets

Definition 1 *([2])*. Let $X = \{x_1, x_2, \ldots, x_n\}$ be a fixed non-empty universe set, an IFS A in X is defined as $A = \{<x, \mu_A(x), \nu_A(x)> | x \in X\}$, which is characterized by a membership function $\mu_A : X \rightarrow [0, 1]$ and a non-membership function $\nu_A : X \rightarrow [0, 1]$ with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$ where μ_A and ν_A represent, respectively, the membership and non-membership degrees of the element x to the set A. In addition, for each IFS A in X, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. $\pi_A(x)$ is denoted the hesitation degree (hesitation margin) of the element x to the set A. Especially, if $\pi_A(x) = 0$, then the IFS A is degraded to a FS.

The complement of an IFS A is defined as $A^c = \{<x, \nu_A(x), \mu_A(x)> | x \in X\}$. For an IFS A, the pair $<\mu_A(x), \nu_A(x)>$ is called an IFV [33]. Denote the set of all IFVs by Ω.

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