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Evolutionary multi-objective optimization algorithms for fuzzy portfolio selection

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ABSTRACT

In this paper, we consider a recently proposed model for portfolio selection, called *Mean-Downside Risk-Skewness* (MDRS) model. This modelling approach takes into account both the multidimensional nature of the portfolio selection problem and the requirements imposed by the investor. Concretely, it optimizes the expected return, the downside-risk and the skewness of a given portfolio, taking into account budget, bound and cardinality constraints. The quantification of the uncertain future return on a given portfolio is approximated by means of LR-fuzzy numbers, while the moments of its return are evaluated using possibility theory. The main purpose of this paper is to solve the MDRS portfolio selection model as a whole constrained three-objective optimization problem, what has not been done before, in order to analyse the efficient portfolios which optimize the three criteria simultaneously. For this aim, we propose new mutation, crossover and reparation operators for evolutionary multi-objective optimization, which have been specially designed for generating feasible solutions of the cardinality constrained MDRS problem. We incorporate the operators suggested into the evolutionary algorithms NSGAII, MOEA/D and GWASF-GA and we analyse their performances for a data set from the Spanish stock market. The potential of our operators is shown in comparison to other commonly used genetic operators and some conclusions are highlighted from the analysis of the trade-offs among the three criteria.

1. Introduction

Financial optimization models for allocating risky assets involve decision-making under uncertainty. Modern portfolio selection theory comes from the mean-variance (MV) model proposed in [1] and the non-linear programming problem involved in this approach has become a classical optimization problem. Since then, most authors have attempted to build optimal solutions for the portfolio selection problem by means of trade-off analysis between two criteria: the maximization of the (mean) expected returns and the minimization of a measure of the variability of outcomes, the risk of the investment. Regarding the second criterion, the Markowitz analysis of risk is applicable when the returns are normally distributed or the utility function to be maximized is quadratic. Otherwise, Arditti and Levy demonstrated in [2] the importance of the role of skewness in discrete time models. But it can be said that the most commonly used measure of the risk is the variance, although other measures have been considered in the literature [3,4]. Overall, the multi-objective optimization nature of the portfolio selection problem is unquestionable and the use of multi-objective optimization techniques has received a great deal of attention for solving these problems [5–10].

In recent years, there has been a growing interest of including information about trading and investors' requirements into the portfolio selection problem, since not all the relevant information for portfolio selection can be obtained just by optimizing returns and risk simultaneously. Thus, some practical constraints have been added to the portfolio selection problem in order to make it more realistic, such as upper and lower bound constraints, allowing assets combinations which respect the investor's wishes, or cardinality constraints limiting the number of assets participating in the portfolios. With the introduction of such constraints, the portfolio optimization problem becomes a constrained multi-objective problem that is NP-hard, and traditional optimization

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methods cannot be used to find efficient portfolios. To overcome this drawback, *evolutionary multi-objective optimization* (EMO) algorithms have been successfully applied for generating solutions of many portfolio selection models [8,11–16]. In general, EMO algorithms are aimed at approximating the set of Pareto optimal or efficient solutions of a multi-objective optimization problem (the so-called *Pareto optimal front*) by applying evolution-based operators to a population of solutions (for more details, see [17,18]). In [15,19], comprehensive literature reviews about the use of EMO algorithms for solving portfolio selection problems are presented, which is a proof of the growing interest on this research field.

Along with the mean and the variance, some authors include the skewness as a criteria for selecting efficient portfolios when a symmetric behaviour of the returns is not achieved. For example, Lai [5] suggested to balance the maximization of the mean and the skewness of the returns and the minimization of the variance. Subsequently, numerous papers have studied the role of skewness in portfolio selection [6,20–22]. With the introduction of skewness, the portfolio selection problem becomes three-dimensional and, in this case, goal programming techniques [5,23,24] or suitable scalarizing (single-objective) approaches [20,25] are usually applied.

Commonly, solving a portfolio selection problem requires two basic components: (i) a suitable approach for quantifying uncertainty of the future returns on a given portfolio, and (ii) an optimization procedure able to provide Pareto optimal portfolios which fit the investor's conditions. Regarding the future returns, classical portfolio selection problems consider the expected return on assets as problem parameters. They are estimated throughout historical data set, usually assuming that the vector of returns on assets is multivariate-normally distributed. However, since the information available in financial markets is often incomplete and, thus, decisions are made under uncertainty, other authors assume that the uncertainty of future return on assets can be quantified by means of fuzzy logic [26,27]. Under this assumption, the multi-objective portfolio selection problem can be solved by using soft computing approaches and fuzzy optimization decision-making techniques [21,28–32].

In this regards, Vercher and Bermudez [22] have recently applied possibility distributions of LR-type fuzzy numbers to quantify the uncertain returns on a given portfolio, whose membership functions are built using sample quantiles information from historical data. Derived from the distribution properties of the investment returns, the authors proposed a multi-objective optimization model for the portfolio selection problem, called possibilistic *Mean-Downside Risk-Skewness* (MDRS) model. The criteria considered in the MDRS model are the maximization of both the mean and the skewness of the future returns, and the minimization of the absolute semi-deviation below the mean as a risk measure (downside risk). The MDRS model also includes budget, upper and lower bound, and cardinality constraints for both the diversification of investment and the restriction of the number of assets that compose the portfolios. Note that Vercher and Bermudez [22] introduced the skewness in order to incorporate a measurement of the asymmetry of the fuzzy return on a given portfolio and to study its role in the possibilistic portfolio selection problem. For this, they proposed solving the constrained multi-objective optimization MDRS problem by using an evolutionary procedure specially designed for generating non-dominated portfolios of two alternative reformulations of the MDRS model. Each of these two reformulations are a bi-objective optimization problem which optimizes two of the objectives of the MDRS model, while the third objective is considered as an additional constraint. With this, the authors analysed the influence of the skewness either as a criteria or as a constraint, and the results obtained supported previous research in this regard: the introduction of the skewness as a goal provokes important changes in the Pareto optimal front of the portfolio selection problem, and consequently in the patterns of investment.

However, the genetic procedure developed in [22] to find efficient portfolios was designed to manage bi-objective optimization problems. Therefore, the possibilistic MDRS model has not been solved as a whole constrained multi-objective optimization problem. This means that, under the aforementioned fuzzy modelling framework, the three criteria (mean and skewness of future returns, and downside risk) have not been optimized alltogether at the same time yet, taking into account all the constraints mentioned. According to this, the main motivation of this paper is twofold. Firstly, our purpose is to solve the possibilistic MDRS model suggested in [22] as a whole constrained three-objective optimization problem. Secondly, we aim at extracting interesting insights about the relationships and trade-offs among the three objectives from an overall multi-objective perspective.

For these two aims, EMO algorithms are particularly useful since they can provide a set of non-dominated portfolios in just one run, from which the objective functions can be analysed from an overall perspective. But applying EMO algorithms to constrained portfolio optimization problems requires a special care for handling the objectives and constraints [19]. Mainly, for the MDRS model, the difficulty comes from the fact that only a limited number of the available assets participate in the portfolios, but not all of them, because of the cardinality constraint. To overcome this, one option can be to consider commonly used genetic operators and, afterwards, to apply several repair mechanisms for the efficient management of the constraints [19]. Alternatively, we propose new mutation, crossover and reparation operators designed ad-hoc for generating feasible portfolios of the MDRS model, which are based on common ideas of evolutionary operators [33,34]. Next, for analysing the MDRS model and for showing the usefulness and potential of the proposed operators, we compare the performance of several EMO algorithms when our operators are used and when traditional operators are considered with repair mechanisms. For this comparison, we consider the *Non-dominated Sorting Genetic Algorithm* (NSGAII) [35], the *Multi-objective Evolutionary Algorithm Based on Decomposition* (MOEA/D) [36] and the *Global Weighting Achievement Scalarizing Function Genetic Algorithm* (GWASF-GA) [37].

The rest of the paper is organized as follows. Section 2 introduces some general concepts of evolutionary multi-objective optimization and fuzzy sets theory, and provides a literature review of previous related works. The possibilistic MDRS portfolio selection problem that we are considering in this work is described in Section 3. Next, Section 4 presents the proposed mutation, crossover and reparation operators for the MDRS model. Subsequently, numerical results for a data set from the Spanish Stock Market are provided in Section 5, in which the proposed operators are compared to commonly used operators in NSGAII, MOEA/D and GWASF-GA. Finally, we conclude and give future research directions in Section 6.

2. Formulation and background concepts

2.1. Evolutionary multi-objective optimization

Multi-objective optimization problems are mathematical programming problems with a vector-valued objective function, which is usually denoted by $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_n(\mathbf{x}))^T$ for a decision vector $\mathbf{x} = (x_1, \ldots, x_N)^T$, where $f_j(\mathbf{x})$ is a real-valued function defined over the feasible set $S \subseteq \mathbb{R}^N$, for every $j = 1, \ldots, n$. Consequently, the decision space belongs to $\mathbb{R}^N$ while the criterion space belongs to $\mathbb{R}^n$, and the multi-objective optimization problem can be stated as follows: