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Comparison of fuzzy logic based models for the multi-response surface problems with replicated response measures

Özlem Türkşen[a,*], Nevin Güler[b]

[a] Ankara University, Faculty of Science, Statistics Department, Tandoğan, Ankara, Turkey
[b] Muğla Sıtkı Koçman University, Faculty of Science, Statistics Department, Kötekli, Muğla, Turkey

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ABSTRACT

A replicated multi-response experiment is a process that includes more than one responses with replications. One of the main objectives in these experiments is to estimate the unknown relationship between responses and input variables simultaneously. In general, classical regression analysis is used for modeling of the responses. However, in most practical problems, the assumptions for regression analysis cannot be satisfied. In this case, alternative modeling methods such as fuzzy logic based modeling approaches can be used. In this study, fuzzy least squares regression (FLSR) and fuzzy clustering based modeling methods, which are switching fuzzy C-regression (SFCR) and Takagi–Sugeno (TS) fuzzy model, are preferred. The novelty of the study is presenting the applicability of SFCR to the multi-response experiment data set with replicated response measures. Three real data set examples are given for application purposes. In order to compare the prediction performance of modeling approaches, root mean square error (RMSE) criteria is used. It is seen from the results that the SFCR gives the better prediction performance among the other fuzzy modeling approaches for the replicated multi-response experimental data sets.

1. Introduction

An experiment is called a multi-response experiment in which the experimental units are wanted to be evaluated with respect to more than one response. The data analysis of such experiments requires a careful consideration because of the multiple response nature of the data. Simultaneous consideration of multiple responses is necessary for building an appropriate approximating model of each unknown response. Multivariate regression analysis is applied for response modeling in the context of multiple response surface methodology (RSM), which is a collection of mathematical and statistical methods for analysis of multi-response surface problems. If responses are uncorrelated and have the same experimental design, multivariate regression analysis becomes a classical regression analysis for modeling of each responses independently. A complete and detailed explanation about multiple RSM is referred to [1–4].

Although the regression analysis is considered as a basic modeling tool for defining the analytical relationship between input and response variables, it cannot be used in some cases. e.g. when the probability assumptions on responses are not justified, or the number of observations is inadequate, or the relationship between input and response variables have complexity and nonlinearity, or there is uncertain information about the data [5]. In fact, there are many cases where observations cannot be known or quantified exactly. One of these cases is multi-response experiments with replicated response measures in which the observed response values are obtained differently for each experiment condition. Generally speaking, the observed response values are uncertain due to the replication and cannot be correctly represented with a single numerical quantity. In such cases, fuzzy logic, which is firstly introduced by Zadeh [6], can be used as a common tool for modeling of the multi-response surface problems.

The fuzzy logic is an extended version of classical logic and can be described as many-valued logic addressing the uncertainty phenomenon. Therefore, the fuzzy logic allows modeling uncertainty associated with vagueness, imprecision and putting this into appropriate mathematical equations. In recent years, some studies have been carried out about modeling of multi-response surface problems in fuzzy framework. In Lai and Chang [7], fuzzy regression models, based on possibility distributions of predicted responses, are first used to model the relations between process parameters and responses. Akbar et al. [8] applied fuzzy approach for modeling dual response surface (DRS) and Bashiri and Ramezani [9] is used fuzzy programming for modeling of multi-response

* Corresponding author. Tel.: +90 3122126720.
E-mail addresses: turksen@ankara.edu.tr (Ö. Türkşen), nguler@mu.edu.tr (N. Güler).

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Table 1
An experimental design for a multi-response experiment with replicated response measures.

No.	Input levels				Responses								
	$\mathbf{X}_1$	$\mathbf{X}_2$	...	$\mathbf{X}_p$	$\mathbf{Y}_1$			...			$\mathbf{Y}_r$		
1	x_{11}	x_{12}	...	x_{1p}	y_{111}	y_{112}	...	y_{11t}	...	y_{1r1}	y_{1r2}	...	y_{1rt}
2	x_{21}	x_{22}	...	x_{2p}	y_{211}	y_{212}	...	y_{21t}	...	y_{2r1}	y_{2r2}	...	y_{2rt}
⋮	⋮	⋮	...	⋮	⋮	⋮	...	⋮	...	⋮	⋮	⋮	⋮
n	x_{n1}	x_{n2}	...	x_{np}	y_{n11}	y_{n12}	...	y_{n1t}	...	y_{nr1}	y_{nr2}	...	y_{nrt}

problem. In the studies of Xie and Lee [10], Prasad and Nath [11], Lu and Antony [12], and Sharma [13], fuzzy models are generated by using Takagi–Sugeno (TS) fuzzy model, called IF-THEN fuzzy-rule base. In Xu and Dong [14], Türkşen [15], and Türkşen and Apaydın [16,17], fuzzy least squares regression (FLSR) is used for modeling of multi-responses. Bashiri and Hosseininezhad [18] proposed a method to constitute a regression model based on replicates of a response and aggregate regression models so that a fuzzy regression model expresses each response. The obtained regression model includes fuzzy coefficients which consider uncertainty in the collected data. Bashiri and Hosseininezhad [19] interested on modeling of unknown response surfaces by using classical approach and fuzzy concept for responses without replicates and responses with some replicates, respectively.

In this paper, multi-response surface problems with replicated response measures are modeled by using FLSR and fuzzy clustering based modeling approaches which are switching fuzzy C-regression (SFCR) and TS fuzzy model. The main purpose of the study is demonstrating the usage of SFCR for modeling of the replicated multi-response experiments. SFCR has the ability of modeling a data set which has more than two different distributions, or modeling the data set with repeated measures of the same response variable. Therefore, the SFCR is thought to be considerably appropriate for response modeling of replicated response measures in multi-response problems. During the modeling by SFCR, the data set is splitted into subsets as the number of replicated responses and a model is obtained for each subset even if the size of data set is small. The paper is organized as follows. Section 2 contains a brief description about multi-response experiments with replicated response measures and modeling. In Section 3, FLSR is defined in detail. In Section 4, fuzzy clustering-based modeling approaches are explained and brief description about SFCR and TS fuzzy model are given. In Section 5, three real data sets are used to illustrate the applicability of fuzzy modeling approaches with comparison results. Finally, conclusion is given in Section 6.

2. Multi-response experiments with replicated measures

Designing a set of experiments, called data gathering, is the first, basic, and necessary step in order to find the most valuable information about the features of the multi-response problem. The design of an experiment in the multi-response case is more complex than in the case of single response. An efficient design for one response may not be efficient for the other responses. Therefore, the choice of a design should be based on a criterion which incorporates measures of efficiency pertaining to all responses in a multi-response situation. Sometimes, the experimental design may be generated by using the replicated measures of multiple responses as given in Table 1. In Table 1, n denotes the number of experimental units and r is the number of response variables which are composed with replicated t measures. And also, each replication is measured for each setting of a group of p input variables.

One of the objectives in a multi-response experiment with replicated response measures is the simultaneous modeling of the behavior of the response variables as a function of the input variables within some region of interest. Suppose that the kth response value at the ith experimental unit is represented by a general model

$$\mathbf{Y}_{ik(.)} = f_k\left(\mathbf{X}_i, \boldsymbol{\beta}\right) + \varepsilon_{ik(.)}, \qquad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, r \qquad (1)$$

where $\mathbf{X}_i$ is the vector $[x_{i1}x_{i2}\ldots x_{ip}]'$ with x_{ij} being the ith level of the jth coded variable ($i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, p$), $\boldsymbol{\beta}$ is a vector of unknown parameters, $\boldsymbol{\varepsilon}_{ik(.)}$ is a random error, and f_k is a function of known form for the kth response and is assumed to be continuous. If f_k is linear in the elements of $\boldsymbol{\beta}$, then the general model in Eq. (1) is transformed to the linear multi-response model which can be written in matrix form as

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \qquad (2)$$

where $\mathbf{Y} = \left[\mathbf{Y}'_1 : \mathbf{Y}'_2 : \ldots : \mathbf{Y}'_\mathbf{r}\right]'$, $\boldsymbol{\beta} = \left[\boldsymbol{\beta}'_1 : \boldsymbol{\beta}'_2 : \ldots : \boldsymbol{\beta}'_\mathbf{r}\right]'$, $\boldsymbol{\varepsilon} = \left[\boldsymbol{\varepsilon}'_1 : \boldsymbol{\varepsilon}'_2 : \ldots : \boldsymbol{\varepsilon}'_\mathbf{r}\right]'$, and $\mathbf{X}$ is the block-diagonal matrix, $\mathbf{X} = \text{diag}(\mathbf{X}_1, \mathbf{X}_2, \ldots, \mathbf{X}_r)$. However, modeling the data set with replicated response measures, as given in Table 1, is not simple since the classical modeling assumptions can be violated and replicated measures of responses may cause uncertainty. In this case, it is necessary to represent the replicated measures without loosing much information at each experimental unit. In the studies of Kim and Lin [20], Köksoy and Muluk [21], Köksoy and Yalçınöz [22], Köksoy [23], and Köksoy and Hocaoğlu [24], the replicated responses are represented with a single quantity, e.g. mean, standard deviation and these are modeled. However, it will be more proper to define a quantity which expresses the uncertainty or vagueness of the nature of the replicated response data. Modeling methods based on fuzzy logic can be used due to its ability of dealing with the uncertainties on the responses.

3. Fuzzy least squares regression

Fuzzy least squares regression (FLSR) is one kind of fuzzy linear regression which is used as alternative method for classical regression analysis to improve parameter estimates. The FLSR is based on the method proposed by Diamond [25]. In order to apply the FLSR to the multi-response data set, the replicated response measures are considered as fuzzy numbers. The design of multi-response experiment with fuzzy observed responses can be shown in Table 2.

Throughout the paper symmetric triangular fuzzy numbers are employed for the sake of simplicity. In order to represent the observed replicated measures of kth response value $\mathbf{Y}_{ik(.)} = [y_{ik1}\ y_{ik2}\ \ldots\ y_{ikt}]$, $i = 1, 2, \ldots, n$, $k = 1, 2, \ldots, r$ as a

Table 2
An experimental design for a multi-response experiment with fuzzy responses.

No.	Input levels				Fuzzy responses			
	$\mathbf{X}_1$	$\mathbf{X}_2$	...	$\mathbf{X}_p$	$\tilde{\mathbf{Y}}_1$	$\tilde{\mathbf{Y}}_2$	...	$\tilde{\mathbf{Y}}_r$
1	x_{11}	x_{12}	...	x_{1p}	$\tilde{Y}_{11}$	$\tilde{Y}_{12}$	...	$\tilde{Y}_{1r}$
2	x_{21}	x_{22}	...	x_{2p}	$\tilde{Y}_{21}$	$\tilde{Y}_{22}$	...	$\tilde{Y}_{2r}$
⋮	⋮	⋮	...	⋮	⋮	⋮	...	⋮
n	x_{n1}	x_{n2}	...	x_{np}	$\tilde{Y}_{n1}$	$\tilde{Y}_{n2}$	...	$\tilde{Y}_{nr}$