Applied Soft Computing 28 (2015) 109–113

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Intuitionistic fuzzy parameterized soft set theory and its decision making

Irfan Deli [a,*], Naim Çağman [b]

[a] Department of Mathematics, Faculty of Arts and Sciences, 7 Aralık University, 79000 Kilis, Turkey
[b] Department of Mathematics, Faculty of Arts and Sciences, Gaziosmanpaşa University, 60250 Tokat, Turkey

ARTICLE INFO

Article history:
Received 21 February 2013
Received in revised form 3 February 2014
Accepted 30 November 2014
Available online 9 December 2014

Keywords:
Soft sets
Fuzzy sets
FP-soft sets
Intuitionistic FP-soft sets
Decision making

ABSTRACT

In this work, we first define intuitionistic fuzzy parameterized soft sets (intuitionistic FP-soft sets) and study some of their properties. We then introduce an adjustable approaches to intuitionistic FP-soft sets based decision making. Finally, we give a numerical example which shows that this method successfully works.

1. Introduction

Many fields deal with the uncertain data which may not be successfully modeled by the classical mathematics, probability theory, fuzzy sets [30], rough sets [26], and other mathematical tools. In 1999, Molodtsov [25] proposed a completely new approach so-called *soft set theory* that is more universal for modeling vagueness and uncertainty.

After Molodtsov's work, Maji et al. [23] introduced several operations of soft sets in more detail. Ali et al. [4] gave some new notions such as the restricted intersection, the restricted union, the restricted difference and the extended intersection of two soft sets and Sezgin and Atagün [28] were extended the theoretical aspect of operations on soft sets. Then, Çağman and Enginoglu [13] redefined the operations of Molodtsov's soft sets to make them more functional for improving several new results and gave products of soft sets and *uni–int* decision function. Also Çağman and Enginoglu [11], defined soft matrices and their operations which are more functional to make theoretical studies in the soft set theory in the problems that contain uncertainties. Majumdar and Samanta [24] and Kharal [20] presented some similarity measures for soft set with applications.

Application of soft sets in algebraic structures is introduced by Aktaş and Çağman [3]. Later the algebraic structure of soft set theory has been studied increasingly (e.g. [1,6,16]) and also algebraic structure of fuzzy soft set theory has been studied by Aygünoglu and Aygün [8].

Some researchers have worked many interesting applications of soft set theory by embedding the ideas of fuzzy sets. Feng et al. [17], Çağman et al. [12] and Yang et al. [29] on theory of fuzzy soft sets, Ali [5] on link between soft sets and fuzzy soft sets, Deng and Wang [27] studied an application of fuzzy soft sets in unknown data in incomplete fuzzy soft sets, Basu et al. [9] has been given to get a balanced solution of a fuzzy soft set based decision making problem. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of rough sets (e.g. [5,18]) and intuitionistic fuzzy sets (e.g. [2,19,21,22]).

The notion of FP (fuzzy parametrizied)-soft sets is given by Çağman et al. [10]. Later the applications of FP-soft set theory have been studied by [14,15]. In this paper, we propose intuitionistic fuzzy parameterized soft sets for dealing with uncertainties that is based on both soft sets and intuitionistic fuzzy sets. Therefore, an approximate function of the intuitionistic fuzzy parameterized soft sets are set valued function and decision obtained by using the operations of soft sets and intuitionistic fuzzy sets that make this sets very convenient and easily applicable in practice. In this paper is organized as follows. We first define intuitionistic fuzzy parameterized soft sets (intuitionistic FP-soft sets), and study their operations and properties. We then introduce a decision making

* Corresponding author. Tel.: +903488142662.
E-mail addresses: irfandeli@kilis.edu.tr (I. Deli), naim.cagman@gop.edu.tr (N. Çağman).

http://dx.doi.org/10.1016/j.asoc.2014.11.053
1568-4946/

method based on intuitionistic FP-soft sets. A numerical example is provided illustrates the effectiveness of the method which is more practical.

2. Preliminary

In this section, we give the basic definitions of soft set theory [25], fuzzy set theory [30], intuitionistic fuzzy set theory [7] and FP-soft set theory [10] that are useful for subsequent discussions.

Definition 1. [25] Let U be a universe, $P(U)$ be the power set of U and E be a set of parameters. A soft set S over U is a set defined by a set valued function S representing a mapping

$$f_S : E \to P(U)$$

that is, it can be written a set of ordered pairs

$$S = \{(x, f_S(x)) : x \in E\}$$

Here, f_S is called approximate function of the soft set S and $f_S(x)$ is called x-approximate value of $x \in E$. The subscript S in the f_S indicates that f_S is the approximate function of S.

Generally, $f_S, f_T, f_V, \ldots$, will be used as an approximate functions of $S, T, V, \ldots$, respectively.

Note that if $f_S(x) = \emptyset$, then the element $(x, f_S(x))$ is not appeared in S.

Example 1. Assume that $U = \{u_1, u_2, u_3, u_4\}$ is the universe contains four houses under consideration in a real estate agent and $E = \{x_1, x_2, x_3, x_4\}$ is the set of parameters, where x_i ($i = 1, 2, 3, 4$) stand for 'safety", "cheap", "technological" and "large garden", respectively.

A customer to select a house from the real estate agent can construct a soft set S that describes the characteristic of houses according to own requests. Assume that $f_S(x_1) = \{u_1, u_2, u_5\}$, $f_S(x_2) = \{u_2, u_4\}$, $f_S(x_3) = \emptyset$, $f_S(x_4) = U$ then the soft-set S is written by

$$S = \{(x_1, \{u_1, u_2, u_5\}), (x_2, \{u_2, u_4\}), (x_4, U)\}$$

Definition 2. [30] Let E be a universe. Then a fuzzy set X over E is a function defined as follows:

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where $\mu_X : E \to [0, 1]$.

Here, μ_X called membership function of X, and the value $\mu_X(x)$ is called the grade of membership of $x \in E$. The value represents the degree of x belonging to the fuzzy set X.

Definition 3. [7] Let E be a universe. An intuitionistic fuzzy set A on E can be defined as follows:

$$A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : \ x \in E\}$$

where, $\mu_A : E \to [0, 1]$ and $\gamma_A : E \to [0, 1]$ such that $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for any $x \in E$.

Here, $\mu_A(x)$ and $\gamma_A(x)$ is the degree of membership and degree of non-membership of the element x, respectively.

If A and B are two intuitionistic fuzzy sets on E, then

i. $A \subset B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\gamma_A(x) \ge \gamma_B(x)$ for $\forall x \in E$
ii. $A = B$ if and only if $\mu_A(x) = \mu_B(x)$ and $\gamma_A(x) = \gamma_B(x)$ $\forall x \in E$
iii. $A^c = \{\langle x, \gamma_A(x), \mu_A(x)\rangle : x \in E\}$
iv. $A \cup B = \{\langle x, max(\mu_A(x), \mu_B(x)), min(\gamma_A(x), \gamma_B(x)\rangle : x \in E\}$,
v. $A \cap B = \{\langle x, min(\mu_A(x), \mu_B(x)), max(\gamma_A(x), \gamma_B(x)\rangle : x \in E\}$,
vi. $A + B = \{\langle x, \mu_X(x) + \mu_Y(x) - \mu_X(x)\mu_Y(x), \gamma_X(x)\gamma_Y(x)\rangle : x \in E\}$,
vii. $A \cdot B = \{\langle x, \mu_A(x)\mu_B(x), \gamma_A(x) + \gamma_B(x) - \gamma_A(x)\gamma_B(x)\rangle : x \in E\}$.

Definition 4. [10] Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters and X be a fuzzy set over E. Then a FP-soft set (f_X, E) on the universe U is defined as follows:

$$(f_X, E) = \{(\mu_X(x)/x, f_X(x)) : x \in E\}$$

where $\mu_X : E \to [0, 1]$ and $f_X : E \to P(U)$ such that $f_X(x) = \emptyset$ if $\mu_X(x) = 0$.

Here f_X called approximate function and μ_X called membership function of FP-soft sets.

Example 2. Assume that $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9\}$ is a universal set and $E = \{a_1, a_2, a_3, a_4\}$ is a set of all parameters. If $X = \{0.7/a_1, 0.5/a_2, 0.6/a_3, 0, 9/a_4\}$ is a fuzzy set over E, then we can write the following FP-soft set:

$$(f_X, E) = \Big\{(0.7/a_1, \{u_2, u_3, u_4, u_5, u_7\}), (0.5/a_2, \{u_4, u_5, u_7\}),$$
$$(0.6/a_3, \{u_1, u_2, u_3, u_4, u_9\}), (0.9/a_4, \{u_1, u_2, u_5, u_7, u_8\})\Big\}$$

3. Intuitionistic FP-soft sets

In this section, we define intuitionistic fuzzy parameterized soft sets (intuitionistic FP-soft sets) and their operations.

Definition 5. Let U be an initial universe, $P(U)$ be the power set of U, E be a set of all parameters and K be an intuitionistic fuzzy set over E. An intuitionistic FP-soft sets $\sqcup_K$ over U is defined as follows:

$$\sqcup_K = \Big\{(\langle x, \alpha_K(x), \beta_K(x)\rangle, f_K(x)) : x \in E\Big\}$$

where $\alpha_K : E \to [0, 1]$, $\beta_K : E \to [0, 1]$ and $f_K : E \to P(U)$ with the property $f_K(x) = \emptyset$ if $\alpha_K(x) = 0$ and $\beta_K(x) = 1$.

Here, the function α_K and β_K called membership function and non-membership of intuitionistic FP-soft set, respectively. The value $\alpha_K(x)$ and $\beta_K(x)$ is the degree of importance and unimportant of the parameter x.

Obviously, each ordinary FP-soft set can be written as

$$\sqcup_K = \Big\{(\langle x, \alpha_K(x), 1 - \alpha_K(x)\rangle, f_K(x)) : x \in E\Big\}$$

Note that the sets of all intuitionistic FP-soft sets over U will be denoted by IFPS(U).

Definition 6. Let $\sqcup_K \in$ IFPS(U). If $\alpha_K(x) = 0$ and $\beta_K(x) = 1$ for all $x \in E$, then $\sqcup_K$ is called a empty intuitionistic FP-soft sets, denoted by $\sqcup_\Phi$.

Definition 7. Let $\sqcup_K \in$ IFPS(U). If $\alpha_K(x) = 1$, $\beta_K(x) = 0$ and $f_K(x) = U$ for all $x \in E$, then $\sqcup_K$ is called universal intuitionistic FP-soft set, denoted by $\sqcup_{\tilde{E}}$.

Example 3. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3\}$ is a set of parameters. If

$$K = \{\langle x_1, 0.2, 0.5\rangle, \langle x_2, 0.5, 0.5\rangle, \langle x_3, 0.6, 0.3\rangle\}$$

and

$$f_K(x_1) = \{u_2, u_4\}, f_K(x_2) = \emptyset, f_K(x_3) = U$$

then an intuitionistic FP-soft set $\sqcup_K$ is written by

$$\sqcup_K = \{(\langle x_1, 0.2, 0.5\rangle, \{u_2, u_4\}), (\langle x_2, 0.5, 0.5\rangle, \emptyset), (\langle x_3, 0.6, 0.3\rangle, U)\}$$

If $L = \{\langle x_1, 0, 1\rangle, \langle x_2, 0, 1\rangle, \langle x_3, 0, 1\rangle, \langle x_4, 0, 1\rangle\}$, then the intuitionistic FP-soft set $\sqcup_L$ is an empty intuitionistic FP soft set.

If $M = \{\langle x_1, 1, 0\rangle, \langle x_2, 1, 0\rangle, \langle x_3, 1, 0\rangle, \langle x_4, 1, 0\rangle\}$ and $f_M(x_1) = U$, $f_M(x_2) = U f_M(x_3) = U$ and $f_M(x_4) = U$, then the intuitionistic FP-soft set $\sqcup_M$ is a universal intuitionistic FP-soft set.