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Fuzzy pricing of geometric Asian options and its algorithm

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ABSTRACT

Owing to the fluctuations of the financial market, input data in the options pricing formula cannot be expected to be precise. This paper discusses the problem of pricing geometric Asian options under the fuzzy environment. We present the fuzzy price of the geometric Asian option under the assumption that the underlying stock price, the risk-free interest rate and the volatility are all fuzzy numbers. This assumption makes the financial investors to pick any geometric Asian option price with an acceptable belief degree. In order to obtain the belief degree, the interpolation search algorithm has been proposed. Some numerical examples are presented to illustrate the rationality and practicability of the model and the algorithm. Finally, an empirical study is performed based on the real data. The empirical study results indicate that the proposed fuzzy pricing model of geometric Asian option is a useful tool for modeling the imprecise problem in the real world.

1. Introduction

A standard option is a financial contract which gives the owner of the contract the right, but not the obligation, to buy or sell a specified asset to a prespecified price (strike price) at a prespecified time (maturity). In fact, standard options all share the characteristics: one underlying asset, the effective starting time is present, only the price of the underlying asset at the option's maturity affects the payoff of the option, whether an option is a call or a put is known when sold, the payoff is always the difference between the underlying asset price and the strike price. On the other hand, exotic options are options that do not share one or more of the characteristics of the standard options. It is well known that there are two main types of exotic options, correlation options and path dependent options. Correlation options are options whose payoffs are affected by more than one underlying asset. Path dependent options are options whose payoffs are affected by how the price of the underlying asset at maturity was reached, the price path of the underlying asset. One particular path dependent option, called Asian option, offers interesting insights on the profitability of investments.

Asian options, whose payoffs depend on the average of the underlying asset price over a prespecified time period, are among the most popular path dependent options traded in both exchanges and over-the counter markets. In fact, Asian options were originally used in 1987 when Banker's Trust Tokyo office used them for calculating average options on crude oil contracts, and give the name Asian option (see Zhang [1]). The main motivation of creating these options is that their averaging feature could reduce the risk of market manipulation of the underlying risky asset near to the maturity date. Since Asian options reduce the volatility inherent in the option, the prices of these options are usually lower than the prices of standard European options. Generally speaking, there are distinct types of European-style Asian options depending on whether the geometric or arithmetic average is taken, whether the average is observed continuously or discretely and whether the strike price is floating or fixed. Obviously, pricing Asian options is more difficult than pricing standard options. Consequently, many methods for pricing Asian options have been developed over the past two decades.

In general, the difficulty of pricing Asian options is that the traditional methods, such as the binomial lattice, the partial differential equation, Fourier transform, Direct integration, Convolution method and Monte Carlo simulation, are inaccurate and impractical. For example, Kemna and Vorst [2] presented an analytic expression for the geometric Asian option, however, a closed-form solution for the arithmetic Asian option has not been found yet. Rogers and Shi [3] reduced the problem of pricing Asian option to the problem of solving a parabolic partial differential equation (PDE) in two variables with the finite difference method. Bénédicte et al. [4] firstly derived a one-state-variable partial differential equation to characterize the valuation of a European type Asian option. The authors also derived some new results on the hedging

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of an Asian option and proposed analytical and numerical analysis on the comparison between Asian and European options. Eric and Alexandre [5] presented an efficient method for pricing discrete Asian options in the presence of smile and non-proportional dividends. They reduced an n-dimensional problem to a one- or two-dimensional one by using a homogeneity property, and examined different numerical specifications of the dimension reduced PDE. Dubois and Lelièvre [6] considered the PDE proposed by Rogers and Shi [3] and proposed a scheme which produced accurate prices for Asian options. Huguette et al. [7] concentrated on the Cox–Ross–Rubenstein model with a daily time step and derived price intervals for arithmetic Asian options with discrete sampling which are forward starting, whose method is based on comonotonicity and on conditioning on the final value of the underlying asset. Fusai and Meucci [8] presented methodologies to price discretely monitored Asian options when the underlying asset evolves according to a generic Lévy process and provided closed-form solutions in terms of the Fourier transform for geometric Asian options. Moreover, the authors also derived a recursive theoretical formula for pricing arithmetic Asian options by recursive integration. Bayraktar and Xing [9] constructed an exponentially fast sequence of functions that uniformly converged to the price of Asian option, which was written on a stock whose dynamics followed the jump-diffusion, and obtained a fast numerical approximation scheme for the pricing problem. Friedrich and Carlo [10] provided a semi explicit valuation formula for geometric Asian options, with fixed and floating-strike under continuous monitoring, when the underlying stock price process exhibited both stochastic volatility and jumps. Recently, Cai and Kou [11] proposed a closed-form solution for Asian options under the hyper-exponential jump-diffusion model.

Although a lot of effort is being spent on improving option pricing model, an efficient and effective method has not to be found yet. A most important factor is that the sources of uncertainty come from not only randomness but also fuzziness. It should be noted that in complicated financial market most of the information on which our decisions are based is linguistic rather than numerical in nature. From this perspective, a new method should be proposed in the problem of pricing options. It is also worth to emphasize that all the papers discussed above assumed that the input variables of the pricing models are regarded as precise real numbers. However, in the real world, the data sometimes cannot be recorded or collected precisely. Especially, the stock price, risk-free interest rate and volatility may be imprecisely determined. For instance, there are many riskless interest rates in financial market, but we do not know whether the riskless interest rate chosen is better or not. Moreover, many financial investors are concerned with the range of options prices, i.e. the prices of options often oscillate within this range. The fuzzy sets theory proposed by Zadeh [12] is a very useful tool for modeling this kind of imprecise problem. The book of collected papers edited by Ribeiro et al. [13] applied the fuzzy set theory to the discipline called financial engineering. Wu [14–16] dealt with the fuzzy European option pricing problems using Black–Scholes pricing formula. Yoshida [17] proposed a new model for pricing European options with uncertainty of both randomness and fuzziness. Lee et al. [18] applied the fuzzy set theory to the Cox–Ross–Rubinstein model to set up the fuzzy binominal European option pricing model which can provide reasonable ranges of options price based on the volatilities of stock prices from greatest to smallest. Thiagarajah et al. [19] considered moment properties for a class of quadratic adaptive fuzzy numbers and obtained the pricing model for European options. Muzzioli et al. [20] obtained a possibility distribution on the risk-neutral probabilities, and performed the risk-neutral valuation of American options as a consequence of the uncertainty in the volatility. Thavaneswaran et al. [21] provided a description of European option price specification errors using the fuzzy weighted possibilistic option valuation model. Xu et al. [22] made an extension of the Merton's normal jump-diffusion model and presented a fuzzy normal jump-diffusion model for pricing European options with uncertainty of both randomness and fuzziness in the jumps. The crisp weighted possibilistic mean normal jump-diffusion model was obtained as well. Yen [23] gave a non-uniform coder in the adaptive neural fuzzy system for option pricing. Zhang et al. [24] proposed the fuzzy pricing formula of American options under the assumption that the price of stock, discount rate, the volatility, and interest rate are all fuzzy numbers. To the best of our knowledge, there are only few papers that studied the problem of pricing exotic options under Fuzzy Environment such as Thavaneswaran et al. [25] and Wang et al. [26]. Recently, Zhan [27] derived the fuzzy patterns of the geometric Asian option pricing formula and the arithmetic Asian option approximate pricing formula by assuming the fuzzy dividend rate, etc., but the author didn't present the algorithm to calculate the corresponding belief degree with a given price. As far as we know, there is no literature research on pricing geometric Asian options under fuzzy environment. The aim of this paper is to fill the gap.

This paper discusses the problem of pricing Asian options under the assumptions that the input parameters in the traditional pricing models are trapezoidal fuzzy numbers. The contribution of this paper is threefold. First, we propose the fuzzy pricing model of Asian call options under the considerations of fuzzy risk-free interest rate, fuzzy volatility and fuzzy stock price. In this situation, the Asian option price turns into a fuzzy number, which makes the financial analyst pick any European option price with an acceptable belief degree for his (her) later use. Second, we design the interpolation search algorithm to solve the fuzzy pricing problem. We also present some numerical examples to illustrate the rationality and practicability of the proposed model and the proposed algorithm. Third, to assess the performance of our fuzzy model and algorithm, we carry out an empirical study using the real data.

The remainder of the paper is organized as follows. In Section 2, we introduce the precise pricing model of Asian call option, which was proposed by Kemna and Vorst [2]. We also propose a general fuzzy pattern of Asian call option pricing model in the latter part of this section. In Section 3, we present specific fuzzy pricing formula under the assumptions that the risk-free interest rate, stock price and its volatility are trapezoidal fuzzy numbers. In Section 4, we design the interpolation search algorithm for the fuzzy pricing model, and give numerical examples to test the effectiveness of the proposed pricing model and algorithm. In Section 4, the empirical study is performed. Finally, some important conclusions and directions for future research are stated in the final Section 6.

2. General fuzzy pricing model of Asian options

Financial products have been weeding through the old to bring forth the new, therefore options have extended many kinds of options called exotic options. Asian options, which are often used for protection against brutal and unexpected changes of prices, represent an important class of exotic options. In fact, Asian options come in various flavors. For example, the average can be arithmetic or it can be geometric. For a plain vanilla Asian option the average is computed over the full trading period, and for a backward-starting option if it is computed over a right subinterval of the trading period. This interval usually has a fixed starting point in time. Moreover, the Asian option can be fixed-strike or floating-strike (if the strike is itself an average). It is called flexible when the payoff is a weighted average, and equally weighted when all the weights are equal. The prices are discretely sampled if the payoff is the average of a discrete set of values of the underlying asset, and