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Interference effects in quantum belief networks☆

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ABSTRACT

Probabilistic graphical models such as Bayesian Networks are one of the most powerful structures known by the Computer Science community for deriving probabilistic inferences. However, modern cognitive psychology has revealed that human decisions could not follow the rules of classical probability theory, because humans cannot process large amounts of data in order to make judgments. Consequently, the inferences performed are based on limited data coupled with several heuristics, leading to violations of the law of total probability. This means that probabilistic graphical models based on classical probability theory are too limited to fully simulate and explain various aspects of human decision making.

Quantum probability theory was developed in order to accommodate the paradoxical findings that the classical theory could not explain. Recent findings in cognitive psychology revealed that quantum probability can fully describe human decisions in an elegant framework. Their findings suggest that, before taking a decision, human thoughts are seen as superposed waves that can interfere with each other, influencing the final decision.

In this work, we propose a new Bayesian Network based on the psychological findings of cognitive scientists. In Computer Science, to the best of our knowledge, there is no quantum like probabilistic system proposed, despite their promising performances. We made experiments with two very well known Bayesian Networks from the literature. The results obtained revealed that the quantum like Bayesian Network can affect drastically the probabilistic inferences, specially when the levels of uncertainty of the network are very high (no pieces of evidence observed). When the levels of uncertainty are very low, then the proposed quantum like network collapses to its classical counterpart.

1. Introduction

The problem of violations of the axioms of probability go back to the early 60s. Ellsberg [32] published a work that influenced modern psychology by showing that humans violate the laws of probability theory when making decisions under risk. The principle that humans were constantly violating is defined by *The Sure Thing Principle*. It is a concept widely used in game theory and was originally introduced by Savage [64]. This principle is fundamental in Bayesian probability theory and states that if one prefers action A over B under state of the world X, and if one also prefers A over B under the complementary state of the world X, then one should always prefer action A over B even when the state of the world is unspecified.

Cognitive psychologists Tversky and Kahneman also explored more situations where classical probability theory could not be accommodated in human decisions. In their pioneering work, Tversky and Kahneman [69] realized that the beliefs expressed by humans could not follow the rules of Boolean logic or classical probability theory, because humans cannot process large amounts of data in order to make estimations or judgments. Consequently, the inferences performed are based on limited data coupled with several heuristics, leading to a violation on one of the most important laws in Bayesian theory: the law of total probability.

One of the key differences between classical and quantum theories is the way how information is processed. According to classical decision making, a person changes beliefs at each moment in time, but it can only be in one precise state with respect to some judgment. So, at each moment, a person is favoring a specific belief. The process of human inference deterministically either jumps between definite states or stays in a single definite state across time [19]. Most computer science, cognitive and decision systems are modeled according to this single path trajectory principle. Fig. 1 illustrates this idea.

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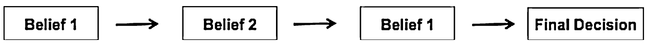

Fig. 1. Example of how information is processed in a classical setting. At each time, beliefs can only be in one definite state.

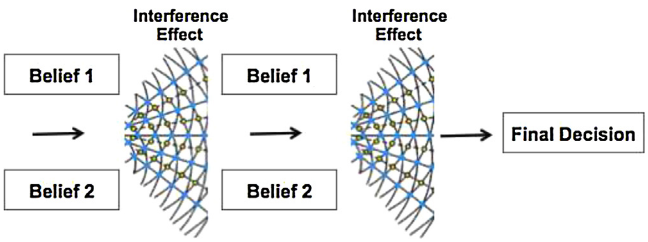

Fig. 2. In human decision making, believes occur in the human mind at the same time, leading to uncertainty feelings and ambiguity. Beliefs can be represented in superposition states that can generate interferences between them.

In quantum information processing, on the other hand, information (and consequently beliefs) are modeled via wave functions and therefore they cannot be in definite states. Instead, they are in an indefinite quantum state called the *superposition* state. That is, all beliefs are occurring on the human mind at the same time. According to cognitive scientists, this effect is responsible for making people experience uncertainties, ambiguities or even confusion before making a decision. At each moment, one belief can be more favored than another, but all beliefs are available at the same time. In this sense, quantum theory enables the modeling of the cognitive system as it was a wave moving across time over a state space until a final decision is made. From this superposed state, uncertainty can produce different waves coming from opposite directions that can crash into each other, causing an interference distribution. This phenomena can never be obtained in a classical setting. Fig. 2 exemplifies this. When the final decision is made, then there is no more uncertainty. The wave collapses into a definite state. Thus, quantum information processing deals with both definite and indefinite states [19].

1.1. Motivation: violations in the two-stage gamblings

Tversky and Shafir [71] were one of the first researchers to test the veracity of Savage's principle under human cognition in a gambling game. In their experiment, participants were asked at each stage to make the decision of whether or not to play a gamble that has an equal chance of winning \$200 or losing \$100. Fig. 3 illustrates the experiment. Three conditions were verified:

1 Participants were informed if they had won the first gamble;
2 Participants were informed if they had lost the first gamble;
3 Participants did not know the outcome of the first gamble.

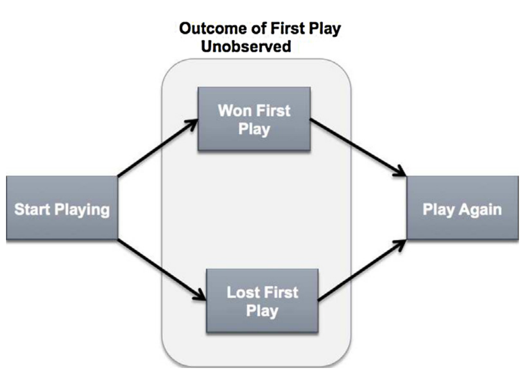

Fig. 3. The two-stage gambling experiment proposed by Tversky and Shafir [71].

The two-stage gambling game was one of the first experiments used in order to determine if the sure thing principle would be verified even with people that did not know about the existence of this principle. The results obtained in Tversky and Shafir [71] experiment showed that this principle is constantly being violated and consequently humans do not perform inferences according to the laws of probability theory and Boolean logic.

The overall results revealed that participants who knew that they won the first gamble, decided to play again. Participants who knew that they lost the first gamble, also decided to play again. Through Savage's sure thing principle, it was expected that the participants would choose to play again, even if they did not know the outcome of the first gamble. However, the results obtained revealed something different. If the participants did not know the outcome of the first gamble, then many of them decided not to play the second one.

Several researchers replicated this experiment. The overall results are specified in Table 1.

Why did the findings reported in Table 1 generate so much controversy in the scientific community? Because, the data observed is not in accordance with the classical law of total probability. In Tversky and Shafir's experiment [71], the probability of a participant playing the second gamble, given that the outcome of the first gamble is unknown, $Pr(G|U)$, can be computed through the law of total probability:

$$Pr(G|U) = Pr(W|U) \cdot Pr(G|W) + Pr(L|U) \cdot Pr(G|L) \quad (1)$$

In Eq. (1), $Pr(W|U)$ corresponds to the probability of a player winning the first gamble, given that (s)he participated on the game in the first place. $Pr(G|W)$ is the probability of playing the second gamble, given that it is known that the player won the first one. $Pr(L|U)$ corresponds to the probability of losing the first gamble, given that the participant decided to play the game in the first place. And finally, $Pr(G|L)$ is the probability of a participant playing the second gamble, given that it is known that (s)he lost the first one.

Following the law of total probability in Eq. (1), the probability of playing the second gamble, given that the player did not know the outcome of the first one, should be between the following values [19]:

$$Pr(G|W) \geq Pr(G|U) \geq Pr(G|L) \quad (2)$$

The findings reported by Tversky and Shafir [71], however, revealed a different relation. Eq. (3) demonstrates that this relation is violating one of the most fundamental laws of Bayesian probability theory:

$$Pr(G|W) = 0.69 \geq Pr(G|L) = 0.58 \geq Pr(G|U) = 0.37 \quad (3)$$

Tversky and Shafir [71] explained these findings in the following way: when the participants knew that they won, then they had extra house money to play with and decided to play the second round. If the participants knew that they lost, then they chose to play again with the hope of recovering the lost money. But, when the participants did not know if they had won or lost the first gamble, then these thoughts, for some reason, did not emerge in their minds and consequently they decided not to play the second gamble. Other works in the literature also replicated this two-stage gambling experiment [65,50,51], also reporting similar results to Tversky and Shafir [71]. Their results are summarized in Table 1.

There have been different works in the literature trying to explain and model this phenomena [19,61,23]. Although the models in the literature diverge, they all agree in one thing: one cannot use classical probability theory to model this phenomena, since the most important rules are being violated. This two stage gambling game experiment was one of the most important works that motivated the use of different theories outside of classical Bayesian