Applied Soft Computing xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

Induced generalized hesitant fuzzy Shapley hybrid operators and their application in multi-attribute decision making

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A R T I C L E I N F O

Article history:
Received 25 March 2013
Received in revised form 23 August 2014
Accepted 18 November 2014
Available online xxx

Keywords:
Multi-attribute decision making
Hesitant fuzzy set
Grey relational analysis (GRA) method
Shapley function

A B S T R A C T

In this study, two induced generalized hesitant fuzzy hybrid operators called the induced generalized hesitant fuzzy Shapley hybrid weighted averaging (IG-HFSHWA) operator and the induced generalized hesitant fuzzy Shapley hybrid geometric mean (IG-HFSHGM) operator are defined. The prominent characteristics of these two operators are that they do not only globally consider the importance of elements and their ordered positions, but also overall reflect their correlations. Furthermore, when the weight information of the attributes and the ordered positions is partly known, using grey relational analysis (GRA) method and the Shapley function models for the optimal fuzzy measures on an attribute set and on an ordered set are respectively established. Finally, an approach to hesitant fuzzy multi-attribute decision making with incomplete weight information and interactive conditions is developed, and an illustrative example is provided to show its practicality and effectivity.

1. Introduction

As an extension of fuzzy sets [1], hesitant fuzzy sets [2] permit the membership degree having a set of possible values. The popularity of hesitant fuzzy sets for solving decision-making problems is that they facilitate effectively representing inherent hesitancy and uncertainty in the human decision making process. Torra [2] discussed the relationship between hesitant fuzzy sets and intuitionistic fuzzy sets and showed that the envelope of a hesitant fuzzy set $h \neq \emptyset$ is an intuitionistic fuzzy set. Based on the relationship between hesitant fuzzy sets and intuitionistic fuzzy sets, Xia and Xu [3] defined some operational laws on hesitant fuzzy sets and presented some hesitant fuzzy aggregation operators. Based on the idea of quasi-arithmetic means, Xia et al. [4] developed a series of hesitant fuzzy aggregation operators and applied them to decision making. Motivated by the idea of the prioritized aggregation operator, Wei [5] developed the hesitant fuzzy prioritized weighted average (HFPWA) operator and the hesitant fuzzy prioritized weighted geometric (HFPWG) operator. Based on the Bonferroni mean (BM), Zhu et al. [6] introduced the weighted hesitant fuzzy geometric Bonferroni mean (WHFGBM) operator. Xu and Xia [7,8] defined some distance measures for hesitant fuzzy sets and presented a distance and correlation measure for hesitant fuzzy information, whilst Xu and Xia [9] studied entropy and cross-entropy of hesitant fuzzy sets and applied them to multi-attribute decision making. Liao and Xu [10] applied the VIKOR-based method to hesitate fuzzy multi-criteria decision making, whilst Chen et al. [11] developed an approach to multi-criteria decision making under hesitant fuzzy environment using the ELECTRE I. Furthermore, Chen et al. [12] studied some correlation coefficients of hesitant fuzzy sets and researched their application in clustering analysis. However, these measures are all defined under the assumption that the values in all HFEs are arranged in an increasing order, and two compared HFEs must have the same length.

To cope with the situation where the elements in a set are correlative, some hesitant fuzzy aggregation operators based on fuzzy measures [13] are defined, such as the weighted hesitant fuzzy Choquet geometric Bonferroni mean (WHFCGBM) operator [6], the hesitant fuzzy Choquet geometric (HFCG) operator [14], the generalized hesitant fuzzy Choquet ordered averaging (GHFCOA) operator and the generalized hesitant fuzzy Choquet ordered geometric (GHFCOG) operator [15]. All these operators are based on the assumption that the fuzzy measure on an attribute set is completely known. However, because of various reasons, such as time pressure and the expert's limited expertise about the problem domain, the weight information in the process of decision making may be partly known. As Meng et al. [16] noted, the Choquet integral only reflects

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http://dx.doi.org/10.1016/j.asoc.2014.11.017
1568-4946/© 2014 Published by Elsevier B.V.

the interaction between two adjacent coalitions. Furthermore, the Choquet integral can either give the importance of elements or that of ordered positions, but it cannot both consider these two aspects.

To cope with these issues, this study presents two induced generalized hesitant fuzzy hybrid Shapley operators, which do not only globally consider the importance of the elements in a set and their ordered positions, but also overall reflect their correlations. Furthermore, when the weight vectors on the attribute set and on the ordered set are partly known, models for the optimal fuzzy measures are respectively established, by which the weight vectors on them can be obtained. As a series of development, a method to hesitant fuzzy multi-attribute decision making with incomplete weight information and interaction conditions is developed. To do these, the rest parts of this paper are organized as follows:

In Section 2, some basic concepts about hesitant fuzzy sets are briefly reviewed, and some existing hesitant fuzzy hybrid aggregation operators are introduced. In Section 3, the induced generalized hesitant fuzzy Shapley hybrid weighted averaging (IG-HFSHWA) operator and the induced generalized hesitant fuzzy Shapley hybrid geometric mean (IG-HFSHGM) operator are defined, and some important cases are examined. In Section 4, based on grey relational analysis (GRA) method and the Shapley function, models for the optimal fuzzy measures on the attribute set and on the ordered set are respectively established. In Section 5, an approach to hesitant fuzzy multi-attribute decision making with incomplete weight information and interaction conditions is developed, and a numerical example is provided to illustrate the developed procedure. The conclusion is made in the last section.

2. Some basic concepts

To cope with the situation where the membership degree of an element has several possible values, Torra [2] introduced the concept of hesitant fuzzy sets.

Definition 1. [2] Let $X=\{x_1, x_2, \ldots, x_n\}$be a finite set, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of [0,1].

To be easily understood, the HFS is expressed by a mathematical symbol

$$E = (< x_i,\ h_E(x_i) > |x_i \in X),$$

where $h_E(x_i)$ is a set of some values in [0,1] denoting the possible membership degrees of the element $x_i \in X$ to the set E. For convenience, Xia and Xu [3] called $h = h_E(x_i)$ a hesitant fuzzy element (HFE). Let H be the set of all HFEs.

Given three HFEs represented by h, h_1 and h_2, Torra [2] defined some operations on them, which can be described as follows:

(1) $h^c = \cup_{r \in h} \{1-r\}$;
(2) $h_1 \cup h_2 = \cup r_{r_1 \in h_1, r_2 \in h_2} \max\{r_1, r_2\}$;
(3) $h_1 \cap h_2 = \cup_{r_1 \in h_1, r_2 \in h_2} \min\{r_1, r_2\}$.

Based on the relationship between HFEs and intuitionistic fuzzy values (IFVs), Xia and Xu [3] defined the following new operations on the HFEs h, h_1 and h_2:

(1) $h^\lambda = \cup_{r \in h} \{r^\lambda\}\ \lambda > 0$;
(2) $\lambda h = \cup_{r \in h} \{1-(1-r)^\lambda\}\ \lambda > 0$;
(3) $h_1 \oplus h_2 = \cup_{r_1 \in h_1, r_2 \in h_2} \{r_1 + r_2 - r_1 r_2\}$;
(4) $h_1 \otimes h_2 = \cup_{r_1 \in h_1, r_2 \in h_2} \{r_1 r_2\}$.

Similar to the order relationship between intuitionistic fuzzy elements, Xia and Xu [3] defined the following score function to rank HFEs.

Definition 2. [3] For a HFE h, $S(h) = (1/\#h)\sum_{r \in h} r$ is called the score function of h, where $\#h$ is the number of the elements in h. For two HFEs h_1 and h_2, if $S(h_1) > S(h_2)$, then $h_1 > h_2$; if $S(h_1) = S(h_2)$, then $h_1 = h_2$.

In some cases, the score function fails to distinguish between two distinct HFEs. For example, let $h_1 = \{0.1, 0.9\}$ and $h_2 = \{0.3, 0.5, 0.7\}$ be two HFEs, then their scores are both equal to 0.5. According to Definition 2, it has $h_1 = h_2$. But they are obviously different. For any HFE h, the averaging deviation function is defined by

$$D(h) = \frac{1}{\#h}\sum_{r \in h}(r - S(h))^2 = \frac{1}{\#h}\sum_{r \in h}\left(r - \frac{1}{\#h}\sum_{r \in h} r\right)^2,$$

where $\#h$ is the number of possible values in h.

In order to increase the distinction of HFEs, the order relationship, for any two HFEs h_1 and h_2, is defined by

If $S(h_1) < S(h_2)$, then $h_1 < h_2$.

If $S(h_1) < S(h_2)$, then $\begin{cases} D(h_1) > D(h_2), & h_1 < h_2 \\ D(h_1) = D(h_2), & h_1 = h_2 \end{cases}$.

In above example, if the improvement method is used to rank $h_1 = \{0.1, 0.9\}$ and $h_2 = \{0.3, 0.5, 0.7\}$, then $h_1 < h_2$ for $S(h_1) < S(h_2)$ and $D(h_1) > D(h_2)$.

To both consider the importance of elements and their ordered positions, Xia and Xu [3] defined two generalized hesitant fuzzy hybrid operators called the generalized hesitant fuzzy hybrid averaging (HFHA) operator and the generalized hesitant fuzzy hybrid geometric (GHFHG) operator, given as in the following definition.

Definition 3. [3] Let h_i $(i = 1, 2, \ldots, n)$ be a collection of HFEs in H, $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector on $\{h_i\}_{i = 1, 2, \ldots, n}$ with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, and $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ be the associated weight vector on the ordered set $N = \{1, 2, \ldots, n\}$ with $\omega_i \in [0,1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

(1) The generalized hesitant fuzzy hybrid averaging (GHFHA) operator is defined by

$$\mathrm{GHFHA}\ (h_1, h_2, \ldots, h_n) = \left(\mathop{\oplus}_{j=1}^{n} \omega_j \dot{h}_{(j)}^{\lambda}\right)^{1/\lambda}$$

$$= \cup_{r_{(1)} \in \dot{h}_{(1)}, r_{(2)} \in \dot{h}_{(2)}, \ldots, r_{(n)} \in \dot{h}_{(n)}} \left(1 - \prod_{j=1}^{n}(1 - r_{(j)}^{\lambda})^{\omega_j}\right)^{1/\lambda},$$

where $\lambda > 0$, $(\cdot)$ is a permutation on the weighted HFEs $nw_i h_i$ $(i = 1, 2, \ldots, n)$ with $\dot{h}_{(j)} = nw_{(j)} h_{(j)}$ being the jth largest value of $nw_i h_i$ $(i = 1, 2, \ldots, n)$, and n is the balancing coefficient.

(2) The generalized hesitant fuzzy hybrid geometric (GHFHG) operator is defined by

$$\mathrm{GHFHG}\ (h_1, h_2, \ldots, h_n) = \frac{1}{\lambda}\left(\mathop{\otimes}_{j=1}^{n} (\lambda \dot{h}_{(j)})^{\omega_j}\right)$$

$$= \cup_{r_{(1)} \in \dot{h}_{(1)}, r_{(2)} \in \dot{h}_{(2)}, \ldots, r_{(n)} \in \dot{h}_{(n)}}$$

$$\times \left(1 - \left(1 - \prod_{j=1}^{n}(1 - (1 - r_{(j)})^{\lambda})^{\omega_j}\right)^{1/\lambda}\right),$$

Please cite this article in press as: F. Meng, et al., Induced generalized hesitant fuzzy Shapley hybrid operators and their application in multi-attribute decision making, Appl. Soft Comput. J. (2014), http://dx.doi.org/10.1016/j.asoc.2014.11.017