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Linguistic continuous ordered weighted distance measure and its application to multiple attributes group decision making

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ABSTRACT

This paper develops a new method for group decision making and introduces a linguistic continuous ordered weighted distance (LCOWD) measure. It is a new distance measure that combines the linguistic continuous ordered weighted averaging (LCOWA) operator with the ordered weighted distance (OWD) measure considering the risk attitude of decision maker. Moreover, it also can relieve the influence of extremely large or extremely small deviations on the aggregation results by assigning them smaller weights. These advantages make it suitable to deal with the situations where the input arguments are represented with uncertain linguistic information. Some of the main properties of the LCOWD measure and different particular cases are studied. The applicability of the new approach is also analyzed focusing on a group decision making problem.

1. Introduction

A multiple attribute group decision making (MAGDM) problem is to find a desirable solution from a finite number of feasible alternatives assessed on multiple attributes by decision makers [1]. The fundamental prerequisite of MAGDM is how to aggregate individual decision maker's preference information on alternatives [2]. Information aggregation is a process which combines all individual decision makers' preferences into an overall one by using a proper aggregation technique. A very useful technique for information aggregation is the OWA operator [3]. It provides a parameterized family of aggregation operators including the maximum, the minimum, the average, and others. Since its appearance, the OWA operator has been studied in a wide range of extensions including the ordered weighted geometric operator [4,5], the generalized OWA operator [6], the induced aggregation operators [7–9], the induced generalized OWA operator [10], the induced correlated aggregating operator [11], the generalized ordered weighted logarithm aggregation operator [12], the generalized multiple averaging operator [13], the power aggregation operator [14], the uncertain aggregation operators [15–17], the induced uncertain aggregation operator [18], the fuzzy generalized aggregation operator [19], the linguistic aggregation operators [20–29], the uncertain linguistic aggregation operators [30–32], the intuitionistic fuzzy aggregation operators [33–41], the interval-valued intuitionistic fuzzy aggregation operators [42,43], the fuzzy linguistic aggregation operators [44,45], the interval-valued fuzzy aggregation operators [46–48], and the hesitant fuzzy aggregation operator [49], etc. Moreover, Yager [50] presented the continuous ordered weighted averaging (COWA) operator where the aggregated arguments are interval numbers rather than finite values. And in [51], Yager and Xu developed the continuous ordered weighted geometric averaging (COWGA) operator. Zhou and Chen [52] generalized the COWA operator and obtained the continuous generalized OWA operator (CGOWA) operator. Wu et al. developed the induced continuous ordered weighted geometric operators [53], Chen and Zhou presented the induced generalized continuous ordered weighted averaging operator [54], Zhang and Xu extended the COWA operator to accommodate uncertain linguistic environment and obtained the linguistic COWA operator (LCOWA) operator [55].

Another very practical technique for information aggregation is the distance measure which is used to compare the alternatives with some ideal results. By this comparison, the alternative closest to ideal results is the best one. Xu and Chen developed an ordered weighted distance (OWD) measure [56] which is the generalization of some widely used distance measure, including the normalized Hamming distance, the normalized Euclidean distance, the normalized geometric distance, the max distance, the median distance, and the minimized distance, etc. Merigó and Gil-Lafuente proposed the ordered weighted averaging distance (OWAD) operator [57] and the Euclidean ordered weighted averaging distance operator [58]. They also introduced the ordered weighted quasi-arithmetic distance operator by combining the ordered weighted quasi-arithmetic mean operator with the normalized Hamming distance [59]. Merigó [60] proposed the probabilistic weighted averaging distance operator that uses probabilities, weighted averages, and distance measures. Merigó et al. [61] investigated the probabilistic ordered weighted averaging distance operator by using a unified model between probabilities and the OWA operator. Merigó and Yager defined a new framework of moving distance aggregation operators [62] including the ordered weighted moving averaging distance and induced ordered weighted moving averaging distance. They generalized the application of distance measures with moving averages by using generalized aggregation operators and obtained the generalized ordered weighted moving averaging distance and the induced generalized ordered weighted averaging distance. Merigó and Casanovas developed several induced distance operators, including the induced ordered weighted averaging distance (IOWAD) operator [63], the induced Enclidean ordered weighted averaging distance (IEOWAD) operator [64] and the induced Minkowski OWA distance operator [65]. They presented the linguistic ordered weighted averaging distance operator [66] which provides a parameterized family of linguistic aggregation operators. Xu developed some fuzzy ordered

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distance measures [67], such as the linguistic ordered weighted distance measure, the uncertain ordered weighted distance measure, the linguistic hybrid weighted distance measure, and the uncertain hybrid weighted distance measure. Szmidt and Kacprzyk [68] defined the four basic distances between the intuitionistic fuzzy sets, including the Hamming distance, the normalized Hamming distance, the Enclidean distance, and the normalized Euclidean distance. Xu and Chen developed some continuous distance measures for intuitionistic fuzzy sets [69], and Zeng and Su proposed an intuitionistic fuzzy ordered weighted distance operator [70]. Xu and Yue investigated the distance measures for the interval-valued intuitionistic fuzzy matrices [71,72]. Xu and Xia presented a class of hesitant distance measures for the hesitant fuzzy sets [73,74].

However, the distance measures of uncertain variables mentioned above are all defined based on the endpoints of interval numbers, which means that distance measures have uniform distributions on the corresponding interval variables. Obviously, it varies among group decision making problems under uncertain environment. Hence it is necessary that the risk attitude of decision makers should be properly considered in group decision making problems. To solve this problem, the continuous interval information aggregation operators will be employed to deal with the interval values. The purpose of this paper is to develop a new distance measure called the linguistic continuous ordered weighted distance (LCOWD) measure based on the LCOWA operator. Some properties and different families of the LCOWD measure are studied. Then the LCOWD measure is generalized and the Quasi LCOWD measure and the Heavy LCOWD measure are obtained.

The prominent characteristic of the LCOWD measure is that it provides a parameterized family of distance operators. The decision makers consider the MAGDM problem more clearly according to their risk attitude in aggregation process. Another advantage of LCOWD measure is that it can relieve the influence of extremely large or extremely small deviations on aggregation results by assigning them smaller weights. Note that these characteristics make it suitable to deal with the situations where the input arguments are represented with uncertain linguistic information.

We also propose a new approach to MAGDM in a group decision making problem by using different types of the LCOWD measures. It shows that the different decision results depend on the different particular type of parameters.

The rest of the paper is organized as follows. In Section 2, we briefly describe some preliminaries. Section 3 presents the LCOWD measure and studies some properties. We also develop some extensions of the LCOWD measure. In Section 4, we present a method for multiple attribute group decision making with the LCOWD measure. In Section 5 an illustrative example is analyzed. In Section 6 we end the paper by summarizing the main conclusions.

2. Preliminaries

In this section, we briefly review the uncertain linguistic variable, the OWA operator, the generalized OWA (GOWA) operator, the COWA operator, the LCOWA operator, the distance measure, and the OWD measure.

2.1. Uncertain linguistic variable and operational laws

Let $S=\{s_\alpha|\alpha=-t,\ldots,-2,-1,0,1,2,\ldots,t\}$ be a linguistic label set with odd cardinality, which requires that the linguistic label set satisfies the following characteristics [74–77]:

(1) The linguistic label set S is ordered: if $s_\alpha, s_\beta \in S$ and $\alpha>\beta$, then $s_\alpha > s_\beta$.
(2) There exists the negation operator: $\mathrm{neg}(s_\alpha)=s_\beta$ such that $\alpha+\beta=0$, where s_α and s_β represent possible values for the linguistic variables and t is a positive integer.

The linguistic label set S is called the linguistic scale. For example, a set of nine labels S can be defined as:

$$S=\{s_{-4}=\mathrm{EL}, s_{-3}=\mathrm{VL}, s_{-2}=\mathrm{L}, s_{-1}=\mathrm{SL}, s_0=\mathrm{M}, s_1=\mathrm{SH}, s_2 =\mathrm{H}, s_3=\mathrm{VH}, s_4=\mathrm{EH}\}.$$

Note that EL = Extremely low, VL = Very low, L = Low, SL = Slightly low, M = Medium, SH = Slightly high, H = High, VH = Very high, EH = Extremely high.

In order to preserve all the given information, the discrete linguistic label set S can be extended to a continuous linguistic label set $\tilde{S}=\{s_\alpha|\alpha\in[-q,q]\}$, where $q(q>t)$ is a sufficiently large positive integer. If $s_\alpha \in S$, we call s_α the original linguistic label, which can be used to evaluate alternatives by the decision makers, otherwise, we call s_α the virtual linguistic label, which only appears in operations.

Definition 1. Suppose that $\tilde{s}=[s_\alpha,s_\beta]=\{x|s_\alpha\le x\le s_\beta\}$, then $\tilde{s}$ is called the uncertain linguistic variable, where $s_\alpha, s_\beta\in\tilde{S}$, s_α, s_β are the lower and upper limits, respectively. Especially, $\tilde{s}$ is called the linguistic variable if $s_\alpha=s_\beta$.

If $\tilde{s}=[s_\alpha,s_\beta]$, $\tilde{s}_1=[s_{\alpha_1},s_{\beta_1}]$ and $\tilde{s}_2=[s_{\alpha_2},s_{\beta_2}]$ are any three uncertain linguistic variables, and $l, l_1, l_2\in[0,1]$, then some operational laws can be defined as follows [31]:

(1) $\tilde{s}_1\oplus\tilde{s}_2=[s_{\alpha_1},s_{\beta_1}]\oplus[s_{\alpha_2},s_{\beta_2}]=[s_{\alpha_1}\oplus s_{\alpha_2}, s_{\beta_1}\oplus s_{\beta_2}]=[s_{\alpha_1+\alpha_2}, s_{\beta_1+\beta_2}]$.
(2) $\tilde{s}_1\oplus\tilde{s}_2=\tilde{s}_2\oplus\tilde{s}_1$.
(3) $l\tilde{s}=l[s_\alpha,s_\beta]=[s_{l\alpha},s_{l\beta}]$.
(4) $l_1\tilde{s}\oplus l_2\tilde{s}=(l_1+l_2)\tilde{s}$.
(5) $l(\tilde{s}_1\oplus\tilde{s}_2)=l\tilde{s}_1\oplus l\tilde{s}_2$.

Let Ω be the set of all uncertain linguistic variables. For convenience, if $s_\alpha\in\tilde{S}$, $I(s_\alpha)$ denotes the subscript of additive linguistic label s_α [76], then we have $I(s_\alpha)=\alpha$.

2.2. The OWA operator

The OWA operator [3] is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum, which has been studied in a wide range of applications [77–82]. It can be defined as follows:

Definition 2. An OWA operator of dimension n is a mapping $\mathrm{OWA}: R^n\to R$ that has an associated weighting vector $\mathbf{w}=(w_1,w_2,\ldots,w_n)$ with $\sum_{j=1}^n w_j=1$ and $w_j\in[0,1]$, such that

$$\mathrm{OWA}(a_1,a_2,\ldots,a_n)=\sum_{j=1}^n w_j b_j, \tag{1}$$

where b_j is the jth largest of the arguments $a_1, a_2, \ldots, a_n$.

Similar to [15], it is possible to distinguish between the descending OWA (DOWA) operator and the ascending OWA (AOWA) operator by using $w_j=w^*_{n-j+1}$, where w_j is the jth weight of the DOWA operator and w^*_{n-j+1} is the jth weight of the AOWA operator. The OWA operator is monotonic, commutative, bounded and idempotent. Other properties could be studied such as different families of the OWA operators, different measures for characterizing the weighting vector and the difference between descending and ascending orders [17,63,79,83–91].

2.3. The GOWA operator

The generalized OWA (GOWA) operator was developed by Yager [6]. It uses generalized mean in the OWA operator, which can be defined as follows:

Definition 3. A GOWA operator is a mapping $\mathrm{GOWA}: R^n\to R$ defined by an associated weighting vector $\mathbf{w}=(w_1,w_2,\ldots,w_n)$ of dimension n, such that $\sum_{j=1}^n w_j=1$ and $w_j\in[0,1]$, and a parameter $p\in(-\infty,\infty)$ and $p\neq0$, according to the following formula:

$$\mathrm{GOWA}(a_1,a_2,\ldots,a_n)=\left(\sum_{j=1}^n w_j b_j^{\,p}\right)^{1/p}, \tag{2}$$

where b_j is the jth largest of $a_1, a_2, \ldots, a_n$.

The GOWA operator is monotonic, commutative, bounded and idempotent [6]. If we consider the possible values of the parameter p in the GOWA operator, we can obtain a group of particular cases.