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Connectivity of covering approximation spaces and its applications on epidemiological issue☆

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ABSTRACT

In this paper, the author proposes some new ideas for the *E*-spread information systems for an epidemic *E*, and takes covering approximation spaces as mathematical models of *E*-spread information systems. By characterizations for connectivity of covering approximation spaces, the author solves the problem: How can one know that an epidemic *E* spreads easily or not easily in a *E*-spread information system? Furthermore, the author gives an example to demonstrate the usefulness, which gives a further application of rough set theory in medical sciences of the above result by logical methods and mathematical methods.

1. Introduction

Would an epidemic spread easily in some crowd? It is an topic gained wide attention in epidemiological studies. Let E denote an epidemic and u, u' be two persons in some crowd. By clinical medical researches and practices, it is possible that E would spread easily between the person u and the person u' if both u and u' have the same disease feature α. Further, it is not easy for a healthy person to be infected by the epidemic E. Based on the above, the following E-spread information system is introduced for our discussion.

Definition 1.1. Let E be an epidemic, U be a crowd, A be a set of some disease features and G be a information granule. A four-tuple (E, U, A, G) is called a E-spread information system if the following conditions are satisfied.

(1) For each person u in the crowd U, there is a disease feature $\alpha \in A$ such that u have the disease feature α.
(2) For each disease feature $\alpha \in A$, there is a person u in the crowd U such that u have the disease feature α.
(3) For each person u in the crowd U and each disease feature α in A, the information granule G shows that whether u has the disease feature α.
(4) For each pair u, u' of persons in the crowd U, the epidemic E spreads easily between u and u' if there is a disease feature $\alpha \in A$ such that both u and u' have the disease feature α.

Further, we define "spread easily" for an epidemic E in a E-spread information system.

Definition 1.2. Let (E, U, A, G) be a E-spread information system. We call that the epidemic E spreads easily in (E, U, A, G) if whenever two persons $u, u' \in U$, there are some persons $u_1, u_2, \cdots, u_n \in U$ such that E spreads easily between the person u_i and the person u_{i+1} for each $i = 1, 2, \cdots, n-1$, where $u_1 = u$ and $u_n = u'$.

Thus, the previous question can be given again in the following form.

Question 1.3. How to judge if an epidemic E spreads easily in a E-spread information system (E, U, A, G)?

In discussion of the above question, it is necessary to analyze data collected from (E, U, A, G). In order to extract useful information hidden in data, many methods in addition to classical logic and classical mathematics have been proposed. Rough-set theory, which was proposed by Pawlak in [27], plays an important role in applications of these methods. Their usefulness has been demonstrated by many successful applications in pattern recognition and artificial intelligence (see [2,4,14–17,19,21,25,28–30,32,37,38,40,44,45,49],

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for example). In particular, rough-set theory is also applied in medical science widely (see [18,26,39], for example). Naturally, it is a significative work to investigate Question 1.3 by using rough-set theory. In the past years, with development of information sciences and computer science, applications of rough-set theory have been extended from Pawlak approximation spaces to covering approximation spaces (see [1,9–11,20,33,36,46,50–56], for example).

Recall that formal concept analysis is a kind of important mathematical tool for conceptual data analysis and knowledge processing [8,41]. In formal concept analysis, the data for analysis are described by formal context (U, A, R), which consists of universe U, attributes set A and relation $R \subseteq U \times A$. Based on the formal context, formal concept analysis has been applied to information retrieval, database management systems, software engineering and other aspects [3,6,12,13,42]. In addition, in rough set theory, the data for analysis are described by information system (U, A, F), which corresponds to the formal context in formal concept analysis and consists of universe U, attributes set A and relation F between U and A. So formal concept analysis and rough set theory are two kinds of complementary mathematical tools for data analysis and data processing [47,48].

In this paper, with the help of formal contexts, we establish some relations between E-spread information systems and covering approximation spaces. By these relations, we take covering approximation spaces as mathematical models of E-spread information systems, and convert investigations from the spread of the epidemic E in E-spread information systems to connectivity for covering approximation spaces. Moreover, we characterizes connectivity of covering approximation spaces by definable subsets of covering approximation spaces. Then, we can obtain the spread of the epidemic E in E-spread information systems by a simple algorithm, which answers Question 1.3 and gives a further application of rough set theory in medical sciences by logical methods and mathematical methods.

2. Preliminaries

Definition 2.1. [[8]]Let U be a finite set of objects, A be a finite set of attributes, and R be a binary relation on $U \times A$ (i.e., $R \subseteq U \times A$).

(1) The triple (U, A, R) is called a formal context.
(2) The formal context (U, A, R) is called to be regular if for each $u \in U$ and each $\alpha \in A$, $uR \neq \emptyset$ and $R\alpha \neq \emptyset$, where $uR = \{\beta \in A : (u, \beta) \in R\}$ and $R\alpha = \{v \in U : (v, \alpha) \in R\}$.

Remark 2.2. Let (U, A, R) be a formal context, $u \in U$ and $\alpha \in A$.

(1) $(u, \alpha) \in R$ means that the object u possesses the attribute α.
(2) It is clear that $(u, \alpha) \in R \Leftrightarrow \alpha \in uR \Leftrightarrow u \in R\alpha$.

E-spread information systems can be denoted by formal contexts. In fact, for a E-spread information system, put the crowd and the set of all disease features in the E-spread information system are the set U of objects and the set A of attributes in a formal context respectively. In addition, for a person u and a disease feature α in the E-spread information system, it is the sufficient and necessary condition such that the person u has the disease feature *alpha* that (u, α) belongs to the binary relation R on $U \times A$. Then (U, A, R) is a formal context.

Definition 2.3. Let (E, U, A, G) be a E-spread information system and let (U, A, R) be a formal context obtained by the above. Then the formal context (U, A, R) is called to be generated by (E, U, A, G).

Lemma 2.4. *If (U, A, R) is a formal context generated by a E-spread information system (E, U, A, G). Then (U, A, R) is regular.*

Proof. Let (U, A, R) be a formal context generated by a E-spread information system (E, U, A, G). We only need to proved that $uR \neq \emptyset$ and $R\alpha \neq \emptyset$ for each person $u \in U$ and each disease feature $\alpha \in A$.

(1) Let a person $u \in U$. By Definition 1.1(1), there is a disease feature $\alpha \in A$ such that u have the disease feature α, i.e., $(u, \alpha) \in R$. By Remark 2.2(2), $\alpha \in uR$. It follows that $uR \neq \emptyset$.
(2) Let a disease feature $\alpha \in A$. By Definition 1.1(2), there is a person $u \in U$ such that u have the disease feature α, i.e., $(u, \alpha) \in R$. By Remark 2.2(2), $u \in R\alpha$. It follows that $R\alpha \neq \emptyset$.

By the above (1) and (2), (U, A, R) is regular. □

Definition 2.5. Let (U, A, R) be a formal context.

(1) For $x, y \in U$, x is called to be closely related to y if there is $\alpha \in A$ such that $(x, \alpha) \in R$ and $(y, \alpha) \in R$.
(2) For $x, y \in U$, x is called to be related to y if there are $u_1, u_2, \cdots, u_n \in U$ such that u_i is closely related to u_{i+1} for each $i = 1, 2, \cdots, n-1$, where $u_1 = x$ and $u_n = y$.
(3) (U, A, R) is called objects related if x is related to y for each pair $x, y \in U$.

Covering approximation spaces is an important generalization of topological spaces, which has the broader applications than topological spaces. And, there are some close relations between covering approximation spaces and formal contexts.

Definition 2.6. [[1]]Let U, the universe of discourse, be a finite set, and $\mathcal{K}$ be a family of non-empty subsets of U.

(1) $\mathcal{K}$ is called a covering of U if U is the union of elements of $\mathcal{K}$.
(2) The pair $(U, \mathcal{K})$ is called a covering approximation space if $\mathcal{K}$ is a covering of U.

Proposition 2.7.

(1) *Let (U, A, R) be a regular formal context. Put $\mathcal{K} = \{R\alpha : \alpha \in A\}$. Then $(U, \mathcal{K})$ is a covering approximation space.*
(2) *Let $(U, \mathcal{K})$ be a covering approximation space, where $\mathcal{K} = \{K_\alpha : \alpha \in A\}$. Give a binary relation R on $U \times A$: $(u, \alpha) \in R$ if and only if $u \in K_\alpha$. Then (U, A, R) is a regular formal context.*

Proof.

(1) Since the formal context (U, A, R) is regular, $R\alpha \neq \emptyset$ for each $\alpha \in A$. On the other hand, for each $u \in U$, $uR \neq \emptyset$, i.e., there is $\alpha \in A$ such that $\alpha \in uR$, hence $u \in R\alpha$ from Remark 2.2(2). So $\mathcal{K}$ is a cover of U consisting of non-empty subsets of U. It follows that $(U, \mathcal{K})$ is a covering approximation space.
(2) It is clear that (U, A, R) is a formal context. For each $\alpha \in A$, $K_\alpha \neq \emptyset$, so there is $u \in U$ such that $u \in K_\alpha$, i.e., $(u, \alpha) \in R$, hence $u \in R\alpha \neq \emptyset$ from Remark 2.2(2). On the other hand, for each $u \in U$, there is $\alpha \in A$ such that $u \in K_\alpha$ since $\mathcal{K}$ is a cover of U, hence $(u, \alpha) \in R$, and so $\alpha \in uR \neq \emptyset$ from Remark 2.2(2). Consequently, (U, A, R) is regular. □

Definition 2.8.

(1) Let (U, A, R) be a regular formal context and let $(U, \mathcal{K})$ be a covering approximation space obtained by Proposition 2.7(1). Then $(U, \mathcal{K})$ is called to be induced by (U, A, R).
(2) Let $(U, \mathcal{K})$ be a covering approximation space and let (U, A, R) be a regular formal context obtained by Proposition 2.7(2). Then (U, A, R) is called to be induced by $(U, \mathcal{K})$.