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A multi-objective programming method for solving network DEA

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ABSTRACT

This study proposes the multi-objective programming (MOP) method for solving network DEA (NDEA) models. In the proposed method, the divisional efficiencies (within an organization) and the overall efficiency of the organization are formulated as separate objective functions in the multi-objective programming model. Compared with conventional DEA where the intermediate processes and products are ignored, this work measures the organization's overall efficiency without neglecting the efficiencies of its subunits. Two case studies demonstrate the proposed NDEA–MOP's utility in measuring the efficiencies of an organization with concerning interactive internal process.

Introduction

Data envelopment analysis (DEA) [1,2] has been widely used in assessing the relative efficiencies of decision-making units (DMUs). The BCC model developed by Banker et al. [1] assesses the relative efficiencies of DMUs by extending the constant-returns-to-scale CCR model [2] to variable returns to scale. Consider n DMUs ($j = 1, \ldots, n$) under assessment. Each DMU consumes m inputs ($i = 1, \ldots, m$) and produces s outputs ($r = 1, \ldots, s$), denoted by $X_{1j}, X_{2j}, \ldots, X_{mj}$ and $Y_{1j}, Y_{2j}, \ldots, Y_{sj}$ respectively. The efficiency of DMU$_k$ can be computed by the BCC and CCR models as follows:

$$\begin{aligned} &\text{Max} \quad E_k = \frac{\sum_{r=1}^{s} u_r Y_{rk} - u_0}{\sum_{i=1}^{m} v_i X_{ik}} \qquad \text{(BCC)} \\ &\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r Y_{rj} - u_0}{\sum_{i=1}^{m} v_i X_{ij}} \le 1, \quad j = 1, 2, \ldots, n \\ &\qquad u_r, v_i \ge \varepsilon \quad r = 1, 2, \ldots, s; \quad i = 1, 2, \ldots, m \\ &\qquad u_0 \text{ unrestricted in sign} \end{aligned} \tag{1}$$

$$\begin{aligned} &\text{Max} \quad E_k = \frac{\sum_{r=1}^{s} u_r Y_{rk}}{\sum_{i=1}^{m} v_i X_{ik}} \qquad \text{(CCR)} \\ &\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r Y_{rj}}{\sum_{i=1}^{m} v_i X_{ij}} \le 1, \quad j = 1, 2, \ldots, n \\ &\qquad u_r, v_i \ge \varepsilon \quad r = 1, 2, \ldots, s; \quad i = 1, 2, \ldots, m \end{aligned} \tag{2}$$

In (1) and (2), the objective function E_k is maximized for every DMU$_k$ individually, where X_{ik} and Y_{rk} are the ith input and rth output of DMU$_k$; u_r, v_i are the weights of the outputs and inputs, respectively; and ε is a small positive value which ensures that all weights are nonnegative. In the BCC model, when the intercept of the production function $u_0 > 0$, the efficiency frontier presents decreasing returns to scale; if $u_0 < 0$, it manifests increasing returns to scale; and when $u_0 = 0$, the model results in the constant returns to scale CCR model.

However, from a decision-making perspective, evaluating a DMU involves examining its performance at the firm level as well as the divisional level. The divisional decision-makers in an organization are expected to cooperate to maximize overall performance. Under such circumstances, conventional DEA methods need enhancement to reflect the collaborative interactions in a DMU.

Conventional DEA usually adopts two types of models in measuring efficiencies: the *aggregation* and *separation* approaches. In the *aggregation* model, the DMU is evaluated as a black box and the internal linking activities are de-emphasized. Consequently we cannot evaluate the performance of individual divisions within the DMU. In the separation model, conversely, each division in a DMU

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is treated as an independent unit and the linking activities between other divisions are totally neglected. Both methods are insufficient for evaluating the efficiencies of the organization's linking processes. Network DEA (NDEA) [4,5] arises due to its competence in evaluating both overall and divisional efficiencies in a unified mechanism while attending to internal interactions within DMUs. Network DEA's main objective is to measure the overall efficiency of a DMU, with the divisional efficiencies as the components.

Numerous studies have been devoted to designing NDEA models. Recently, Tone and Tsutsui [6] proposed a general slack-based network DEA approach, Network SBM (NSBM), which can evaluate intermediate products formally. Cook et al. [7] review a special type of two-stage network DEA model where all of the first-stage outputs are the only inputs to the second stage. They additionally classify all methods in two-stage DEA into Stackelberg and cooperative methods. Further Cook et al. [8] examine the general problem of an open multistage process by presenting the overall efficiency as an additive weighted average of the efficiencies of the individual components or stages. In response to Tone and Tsutsui's work [6], Fukuyama and Mirdehghan [9] propose a network DEA approach for identifying the efficiency status of each DMU and its divisions so that inappropriate decisions due to multiple optima can be avoided.

This study intends to propose an alternative approach to network DEA problems via the multi-objective programming (MOP) method. Multiple objective programming is characterized by a set of objective functions that must be optimized simultaneously and a set of well-defined constraints to be satisfied [10]. Zimmermann [11] develops the fuzzy approach by searching for the optimal solution with the highest degree of membership (satisfaction) in the feasible region. Lee and Li [12] later extend Zimmermann's work to fuzzy multiple objective programming. Evolutionary algorithms have recently become a widely used methodology in MOP. Gen et al. [13] apply the multi-objective genetic algorithm (GA) approach to network models and optimization. Shayanfar et al. [14] have developed a new hybrid algorithm incorporating game theory and the genetic algorithm method to address generation expansion planning problems in the energy decision-making of investments. Later, Gitizadeh and Aghaei [15] utilized a multi-objective genetic algorithm (MOGA) to solve the multi-objective electricity energy market-clearing problem. Particle swarm optimization (PSO) and its improved versions have been implemented for MOP by introducing fuzzy sets [16,17], combining stochastic optimization [18] and the shuffled frog-leaping (SFL) algorithm [19], designing a new mutation method to improve global searching ability [20], etc. Other attempts to solve MOP include the stochastic multi-objective framework [21,22], lexicographic optimization [23,24], the honey-bee mating optimization algorithm [25], and so on.

The purpose of this study is to propose the NDEA–MOP as an alternative approach to processing network DEA. Based on the CCR and BCC models, the overall as well as divisional efficiencies within a DMU are defined as separate objective functions to be optimized cohesively. The case studies of electric power companies [6] and the solar energy industry are presented to demonstrate the usefulness of NDEA–MOP. The remainder of this paper is organized as follows: "Model formulation" section develops the NDEA–MOP based on BCC and CCR and the solution procedure based on the fuzzy approach [11]. In "Case studies" section, we present the case studies and compare the results with the related research. The conclusions are given in last section.

Model formulation

This work follows a part of Tone and Tsutsui's notations [6]. Consider n DMUs ($j=1,\ldots,n$) consisting of K divisions ($k=1,\ldots,K$). Let m_k and r_k be the numbers of inputs to and outputs from Division k, respectively. We denote the link streaming from Division k to Division h by (k, h) and the set of links by L. The observed input resources to DMU$_j$ at Division k are $\{\mathbf{x}_j^k \in R_+^{m_k}\}$ ($j=1,\ldots,n$; $k=1,\ldots,K$); the output products from DMU$_j$ at Division k are $\{\mathbf{y}_j^k \in R_+^{r_k}\}$ ($j=1,\ldots,n$; $k=1,\ldots,K$); the linking intermediate products from Division k to Division h are $\{\mathbf{z}_j^{(k,h)} \in R_+^{t_{(k,h)}}\}$ ($j=1,\ldots,n$; $(k,h)\in L$) where $t_{(k,h)}$ is the number of items in Link (k, h). This study develops the cooperative multi-objective programming models for evaluating a general network structure formulated with the NDEA–MOP models.

The NDEA–MOP model

Two types of the NDEA–MOP can be developed by BCC and CCR as follows:

BCC–MOP

$$\text{Max}\quad E_o\quad (\text{DMU : firm level})$$

$$\text{Max}\quad E_o^k=\frac{\sum_{r=1}^{r_k}u_r^kY_{ro}^k+\sum_{\forall(k,h)}\sum_{p=1}^{t_{(k,h)}}\mu_h^kZ_{op}^{(k,h)}-\alpha_k}{\sum_{i=1}^{m_k}v_i^kX_{io}^k+\sum_{\forall(g,k)}\sum_{q=1}^{t_{(g,k)}}\omega_g^kZ_{oq}^{(g,k)}}$$
$$k=1,\ldots,K\quad(\text{divisional level})$$

s.t.
$$\frac{\sum_{r=1}^{r_k}u_r^kY_{rj}^k+\sum_{\forall(k,h)}\sum_{p=1}^{t_{(k,h)}}\mu_h^kZ_{jp}^{(k,h)}-\alpha_k}{\sum_{i=1}^{m_k}v_i^kX_{ij}^k+\sum_{\forall(g,k)}\sum_{q=1}^{t_{(g,k)}}\omega_g^kZ_{jq}^{(g,k)}}\leq 1 \tag{3}$$
$$j=1,2,\ldots,n;\ k=1,2,\ldots,K$$

$$u_r^k, v_i^k, \mu_h^k, \omega_g^k\geq\varepsilon>0,\qquad \alpha_k\quad \text{unrestricted in sign},\ k=1,2,\ldots,K$$
$$r=1,2,\ldots,r_k;\ i=1,2,\ldots,m_k;\ \text{all}\ (k,h),(g,k)\in L$$

CCR–MOP

$$\text{Max}\quad E_o\quad (\text{DMU : firm level})$$

$$\text{Max}\quad E_o^k=\frac{\sum_{r=1}^{r_k}u_r^kY_{ro}^k+\sum_{\forall(k,h)}\sum_{p=1}^{t_{(k,h)}}\mu_h^kZ_{op}^{(k,h)}}{\sum_{i=1}^{m_k}v_i^kX_{io}^k+\sum_{\forall(g,k)}\sum_{q=1}^{t_{(g,k)}}\omega_g^kZ_{oq}^{(g,k)}}$$
$$k=1,\ldots,K\quad(\text{divisional level})$$

s.t.
$$\frac{\sum_{r=1}^{r_k}u_r^kY_{rj}^k+\sum_{\forall(k,h)}\sum_{p=1}^{t_{(k,h)}}\mu_h^kZ_{jp}^{(k,h)}}{\sum_{i=1}^{m_k}v_i^kX_{ij}^k+\sum_{\forall(g,k)}\sum_{q=1}^{t_{(g,k)}}\omega_g^kZ_{jq}^{(g,k)}}\leq 1 \tag{4}$$
$$j=1,2,\ldots,n;\ k=1,2,\ldots,K$$

$$u_r^k, v_i^k, \mu_h^k, \omega_g^k\geq\varepsilon>0,\ r=1,2,\ldots,r_k;$$
$$i=1,2,\ldots,m_k;\ \text{all}\ (k,h),(g,k)\in L$$

In the NDEA–MOP models, the objective function E_o^k measures the efficiency of Division k at DMU$_o$, where the weighted links outgoing from Division k, $\mu_h^kZ_{jp}^{(k,h)}$, $\forall(k,h)$, $p=1,\ldots,t_{(k,h)}$, are regarded as the (intermediate) outputs of Division k and the incoming inputs to Division h, $\omega_k^hZ_{jq}^{(k,h)}$. The overall efficiency E_o of DMU$_o$ is defined as the convex combination $E_o=\sum_{k=1}^{K}w_kE_o^k$ of the K efficiency scores, where w_k denotes the weight representing the relative contribution of division k. The two types of NDEA–MOP in (3) and (4) are designed as cooperative models; that is, the strategic resources are allocated collaboratively by each division as well as at the enterprise decision level.

Solution process

Based on the fuzzy approach proposed by Zimmermann [11], the following algorithm is developed to solve the model in (3)