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A solid transportation problem with type-2 fuzzy variables

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ABSTRACT

This paper focuses on generating the optimal solutions of the solid transportation problem under fuzzy environment, in which the supply capacities, demands and transportation capacities are supposed to be type-2 fuzzy variables due to the instinctive imprecision. In order to model the problem within the framework of the credibility optimization, three types of new defuzzification criteria, i.e., optimistic value criterion, pessimistic value criterion and expected value criterion, are proposed for type-2 fuzzy variables. Then, the multi-fold fuzzy solid transportation problem is reformulated as the chance-constrained programming model with the least expected transportation cost. To solve the model, fuzzy simulation based tabu search algorithm is designed to seek approximate optimal solutions. Numerical experiments are implemented to illustrate the application and effectiveness of the proposed approaches.

Introduction

Traditional transportation problem (TP), which only considers two types of constraints associated with sources and destinations, is one of the well-known network optimization problems. In practice, as the conveyance capacity is another critical parameter in transportation activities, numerous researchers have concentrated their studies on the solid transportation problem by simultaneously considering restrictions of sources, destinations and conveyances. Clearly, the traditional transportation problem is a special case of the solid transportation problem if only one type of conveyance is taken into account. With this concern, the solid transportation problem can be regarded as a generalization of the traditional transportation problem (see Haley [1,2] for the differences between these two problems).

Due to the complex environment during the transportation activities, some critical parameters in the solid transportation problem are always treated as uncertain variables to meet the practical situations. For instance, if one needs to make a transportation plan for the next month, the supply capacity at each source, the demand at each destination and the conveyance capacity are often required to be estimated by professional judgments or probability statistics because of no precise a priori information. In this case, it is more suitable to investigate this problem by using fuzzy or stochastic optimization methodologies. (The relevant theoretical methods can be referred to probability theory and fuzzy set theory, which can also be included in the framework of Granular Computing proposed by Pedrycz [23,24].) Up to now, a variety of researches in literature have handled the solid transportation problems under the uncertain environments. For instance, Jiménez and Verdegay [3] formulated a balanced fuzzy solid transportation problem by considering fuzzy supplies, fuzzy demands and fuzzy conveyance capacities. Liu [4] investigated a non-balanced solid transportation problem in condition that the supply capacities, demands, and conveyance capacities are all convex fuzzy numbers. Yang and Liu [5] proposed the fuzzy programming models for the fixed charge solid transportation problem by assuming that the direct costs, fixed charges, supplies, demands and conveyance capacities are fuzzy variables. Kundu et al. [6] discussed a multi-objective multi-item solid transportation problem with fuzzy coefficients. Ojha et al. [7] treated the source availabilities, destination demands and conveyance capacities as different types of fuzzy numbers, such as general, trapezoidal and triangular fuzzy numbers. Molla et al. [8] investigated a fixed charge solid transportation problem under a fuzzy environment and employed three meta-heuristics to solve the problem. Yang and Feng [9] studied a bi-criteria solid transportation problem, where the parameters are random variables, and designed a hybrid algorithm to generate an approximate optimal solution. Ojha et al. [10] studied a stochastic discounted multi-objective solid transportation problem for breakable items by using analytical hierarchy process.

In reality, uncertainty may exist with multi-fold. For example, in estimating the membership degree of a fuzzy demand, it is not easy to deduce a crisp membership function because of the instinctive imprecision. To handle this case, the type-2 fuzzy set theory was introduced

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by Zadeh [11] in 1975, and further improved by Liu and Liu [12] in 2007 through using the credibility theory. As for the further development of type-2 fuzzy theory, Qin et al. [13] presented three kinds of critical values for a regular fuzzy variable and proposed different methods for the reduction of a type-2 fuzzy variable; Coupland and John [14] presented a new method based on geometric representations and operations for defuzzifing a type-2 fuzzy set; Wu and Mendel [27] defined the concepts of uncertainty measures (including cardinality, fuzziness, variance and skewness) for interval type-2 fuzzy sets; Kundu et al. [25] discussed two fixed charge transportation problems with type-2 fuzzy variables; Greenfield et al. [26] provided a novel approach to improve the speed of defuzzification for discretized generalized type-2 fuzzy sets.

For the solid transportation problem with fuzzy information, we have two motivations to explore this problem within the framework of type-2 fuzzy set theory. Firstly, it is more general and commonsense to treat some critical parameters as type-2 fuzzy variables because of the practical difficulties of determining their crisp membership functions. Secondly, when some parameters are assumed to be type-2 fuzzy variables, designing an effective method to handle the optimization problem is also a challenging issue. With this concern, we are particularly interested in how to formulate the transportation model, and then design effective algorithms to produce the optimal transportation strategies. To this end, this study proposes three new defuzziness methods for type-2 fuzzy variables based on the credibility measure, and then formulates the problem as a chance-constrained expected value programming model. Fuzzy simulation based tabu search algorithm is also designed to seek an approximate optimal solution for the model. Numerical experiments show the application and efficiency of the proposed approaches.

This paper is organized as follows. In Section “Problem statements”, we present a detailed description for the solid transportation problem. In Section “Type-2 fuzzy variables”, we introduce some fundamental knowledge with respect to type-2 fuzzy variables. Then, three types of defuzzification methods are proposed for type-2 fuzzy variables in Section “Defuzzification methods for type-2 fuzzy variables”, where we deduce the possibility distributions of the reduced type-1 fuzzy parameters for type-2 trapezoidal fuzzy variables. Section “Reformulation of the solid transportation problem with type-2 fuzzy variables” reformulates the solid transportation problem with type-2 fuzzy parameters as a chance-constrained expected value model. In Section “Numerical experiments”, several numerical examples are implemented to illustrate the efficiency of the model and algorithm. Finally, a conclusion is made in the last section.

Problem statements

For the completeness of this research, we shall give a detailed description for the solid transportation problem (STP) in this section. As an extension of the traditional transportation problem which only considers two factors (i.e., source and destination), the STP is related to how to transport products from sources to destinations by conveyances so as to minimize the total transportation cost. For any STP, when the total supply of products, the total demand of products and the total transportation capacity of conveyances are equal to each other, we call such a problem as a balanced solid transportation problem. However, when the balance condition cannot be established, we have to investigate the non-balanced solid transportation problem, in which it is required that the total available resources and the total capacities of conveyances are not less than the total demands of destinations. The non-balanced solid transportation problem can be formulated as follows:

$$\begin{array}{ll} \min & Z = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K} c_{ijk}x_{ijk} \\ st. & \sum_{j=1}^{J}\sum_{k=1}^{K} x_{ijk} \le a_i, \quad i = 1, 2, \ldots, I \\ & \sum_{i=1}^{I}\sum_{k=1}^{K} x_{ijk} \ge b_j, \quad j = 1, 2, \ldots, J \\ & \sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijk} \le e_k, \quad k = 1, 2, \ldots, K \\ & x_{ijk} \ge 0, \qquad \forall\, i, j, k, \end{array} \tag{1}$$

where the meanings of notations are listed as follows: $i(=1, 2, \ldots I)$ is the index of source; $j(=1, 2, \ldots J)$ is the index of destination; $k(=1, 2, \ldots K)$ is the index of conveyance; a_i is the total supply of products in source i; b_j is the total demand of products in destination j; e_k is the total transportation capacity of conveyance k; c_{ijk} is the unit transportation cost from source i to destination j by conveyance k; x_{ijk} is the decision variable, which represents the transportation quantity from source i to destination j by conveyance k.

In this model, the first constraint states that the total amount of products transported from source i are no more than the total supply in source i; the second constraint implies that the total amount of products transported from different sources should be greater than the demands of destination j; the third constraint requires that the total amount of products transported from different sources to different destinations by conveyance k are not greater than its transportation capacity. In addition, an implicit non-balance constraint for this model is $\sum_{i=1}^{I} a_i \ge \sum_{j=1}^{J} b_j$ and $\sum_{k=1}^{K} e_k \ge \sum_{j=1}^{J} b_j$. Apparently, the formulated non-balanced solid transportation problem is a linear programming model.

Fig. 1 is given to clearly illustrate the transportation process in solid transportation problems, in which the products are required to be transported from two sources to three destinations by three types of conveyances. Thus, the aim of this problem is to generate an optimal transportation plan from source nodes to destination nodes by different conveyances (i.e., train, truck and ship). In transportation planning process, three constraints need to be considered. (1) For destination j (say node 1), the total amount of products transported from source 1 and source 2 by trains and ships should be greater than the demands of this node. (2) For source i (say node 1), the total amount of products, which are transported to three destinations by three types of conveyances, should be less than the supply capacities of this source. (3) For conveyance trains, when we transport products from sources 1 and 2 to destinations 1 and 3, the total amount of products should not be beyond the transportation capacities of trains, and the similar constraints are also applicable to ships and trucks.

The above mathematical model is formulated with certain parameters taking fixed values, which can be effectively solved by some heuristics, for instance, tabu search algorithm (Yang and Feng [9]), analytical hierarchy process (Ojha et al. [10]), genetic algorithm (Vignaux and Michalewicz [15]), etc. However, in reality, it is difficult to figure out the crisp values of some parameters due to the complexity of the decision environment and the incompleteness of a prior information. In this case, the relevant parameters can be endowed with

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