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Linguistic group decision making with induced aggregation operators and probabilistic information

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ABSTRACT

A new approach for linguistic group decision making by using probabilistic information and induced aggregation operators is presented. It is based on the induced linguistic probabilistic ordered weighted average (ILPOWA). It is an aggregation operator that uses probabilities and OWA operators in the same formulation considering the degree of importance that each concept has in the formulation. It uses complex attitudinal characters that can be assessed by using order inducing variables. Furthermore, it deals with an uncertain environment where the information cannot be studied in a numerical scale but it is possible to use linguistic variables. Several extensions to this approach are presented by using moving averages and Bonferroni means. The applicability of this approach is also studied with a focus on multi-criteria group decision making by using multi-person aggregation operators in order to deal with the opinion of several experts in the analysis. An illustrative example regarding importation strategies in the administration of a country is developed.

1. Introduction

The available information in an environment can be assessed in different ways. Usually, people try to represent the information by using a numerical scale. However, this framework cannot be used always since we may find uncertain environments where the information is very imprecise and it is not possible to represent it in a quantitative way. Therefore, it is necessary to use another framework such as the use of linguistic variables that represent the information in a qualitative way. By using linguistic information we can represent expressions such as *high*, *low*, *very high* and so on. The classic approach for representing linguistic information by using fuzzy sets [1] was introduced by Zadeh [2]. Since its introduction, it has been studied by a lot of authors [3–5]. A very useful approach has been introduced by Xu [5] where he extends the model into a continuous setting. Thus, it is easier to deal with the information without losing information, especially when using operations in the analysis.

Linguistic information has been used in a wide range of decision making problems. For example, Xu [5,6] and Wei [7] studied linguistic decision making problems by using group information. Merigó and Casanovas [8] analyzed a model by using distance measures that was later extended by Zeng and Su [9]. Merigó et al. [10] studied the use of the Dempster–Shafer belief structure. Herrera et al. analyzed a model that used unbalanced linguistic information [3]. Xu and Wang [11,12] introduced a model by using power averages. Xu et al. [13] developed a similar approach when dealing with the 2-tuple linguistic methodology. Xu et al. [14] have designed different scales for dealing with interactive approaches in linguistic decision making.

When dealing with decision making problems [15,16], it is necessary to aggregate the available information in order to make decisions. A very useful aggregation operator is the ordered weighted average (OWA) [17]. It provides a parameterized family of aggregation operators between the minimum and the maximum. The reordering process of the OWA operator can be generalized by using order inducing variables obtaining the induced OWA (IOWA) operator [18]. The OWA and the IOWA operator have also been studied under linguistic environments forming the linguistic OWA (LOWA) [19] and the induced LOWA (ILOWA) operator [6]. These

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operators have been extended by using generalized and quasi-arithmetic means [9,20].

Recently, Merigó [21] has introduced the probabilistic OWA (POWA) operator. It unifies probabilistic aggregations and the OWA operator in the same formulation and considering the degree of importance that each concept has in the analysis. Thus, we are able to consider decision making problems with probabilities and with the attitudinal character of the decision maker. Note that some previous studies already studied the use of the OWA operator in the probability such as the concept of immediate probability [22–24]. Moreover, other authors have developed similar approaches by using the weighted average including the probabilistic weighted average [25], the weighted OWA (WOWA) operator [26], the hybrid average [27] and the importance OWA operator [28,29].

The aim of this paper is to analyze decision making problems under linguistic environments where it is possible to use probabilities and the attitudinal character of the decision maker. For doing so, it introduced the induced linguistic POWA (ILPOWA) operator. It is an aggregation operator that unifies the OWA operator and the probability in the same formulation under a linguistic environment. Its main advantage is that it can deal with probabilistic information and under or overestimate it according to the attitudinal character of the decision maker. This issue is important because the probabilities provide a neutral expectation of the future but cannot guarantee that it is the correct one. Thus, sometimes we may find situations where the results tend to be higher or lower than those provided by the probabilities. Some of its main properties and particular cases are studied. Several extensions are also developed by using moving averages forming the induced linguistic probabilistic ordered weighted moving average (ILPOWMA). Furthermore, it also presented the use of Bonferroni means in the ILPOWA operator. Note that the main contribution of this paper is the integration of OWA aggregation operators with probabilities under linguistic environments being the first POWA model that uses linguistic information in the aggregation process.

This approach is studied in a linguistic multi-criteria group decision making problem by using multi-person aggregation operators. Thus, it formed the multi-person ILPOWA (MP-ILPOWA) operator. It is an aggregation operator that deals with the opinion of several experts in the analysis when this information is provided in the form of linguistic variables. Several particular cases are studied including the multi-person linguistic probabilistic aggregation (MP-LPA), the multi-person LOWA (MP-LOWA) and the multi-person linguistic arithmetic mean (MP-LAM). An illustrative example is also presented focused on a linguistic decision making application regarding the importation strategy of a country.

The rest of the paper is organized as follows. Section 2 reviews some basic preliminaries. Section 3 presents the ILPOWA operator and some basic families. Section 4 develops several extensions by using moving averages and Bonferroni means. Section 5 analyzes the new linguistic group decision making approach and the MP-ILPOWA operator. Section 6 summarizes the main results of the paper.

2. Preliminaries

In this section, we briefly review the linguistic approach and some basic aggregation operators including the OWA, the linguistic OWA, the induced OWA and the probabilistic OWA operator.

2.1. The linguistic approach

Usually, people are used to work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of the real world cannot be assessed in a quantitative form. Instead, it is possible to use a qualitative one, i.e., with vague or imprecise knowledge such as the use of linguistic assessments instead of numerical values. The linguistic approach represents qualitative aspects as by means of linguistic variables [2].

We have to select the appropriate linguistic descriptors for the term set and their semantics. One possibility for generating the linguistic term set consists in directly supplying the term set by considering all terms distributed on a scale on which a total order is defined. For example, a set of eleven terms S could be given as follows:

$$S=\{s_0=N,\ s_1=EL,\ s_2=VL,\ s_3=L,\ s_4=LM,\ s_5=M,\ s_6=MH,\ s_7=H,\ s_8=VH,\ s_9=EH,\ s_{10}=P\}.$$

Note that $N=None$, $EL=Extremely\ low$, $VL=Very\ low$, $L=Low$, $LM=Low\text{-}Medium$, $M=Medium$, $MH=Medium\text{-}High$, $H=High$, $VH=Very\ high$, $EH=Extremely\ high$, $P=Perfect$. Usually, in these cases, it is required that in the linguistic term set there exists:

- A negation operator: $\mathrm{Neg}(s_i)=s_j$ such that $j=g+1-i$.
- The set is ordered: $s_i \le s_j$ if and only if $i \le j$.
- Max operator: $\mathrm{Max}(s_i, s_j)=s_i$ if $s_i \ge s_j$.
- Min operator: $\mathrm{Min}(s_i, s_j)=s_i$ if $s_i \le s_j$.

Different approaches have been developed for dealing with linguistic information such as [2–5]. In this paper, we follow the ideas of Xu [5,6]. Thus, in order to preserve all the given information, we extend the discrete linguistic term set S to a continuous linguistic term set $\hat{S}=\{s_\alpha | s_1 < s_\alpha \le s_t,\ \alpha \in [1,t]\}$, where, if $s_\alpha \in S$, we call s_α the original linguistic term, otherwise, we call s_α the virtual linguistic term.

Consider any two linguistic terms $s_\alpha, s_\beta \in \hat{S}$, and $\mu, \mu_1, \mu_2 \in [0,1]$, we define some operational laws as follows [5,6]:

- $\mu s_\alpha = s_{\mu\alpha}$.
- $s_\alpha + s_\beta = s_\beta + s_\alpha = s_{\alpha+\beta}$.
- $(s_\alpha)^\mu = s_{\alpha\mu}$.
- $s_\alpha \times s_\beta = s_\beta \times s_\alpha = s_{\alpha\beta}$.

Note that this model is very useful for computing with words because it is very easy to use and it follows a similar methodology as the numerical information.

2.2. The OWA operator

The OWA operator [17] is an aggregation operator that provides a parameterized family of aggregation operators between the minimum and the maximum. It is very useful in decision making under uncertainty since it allows the use of decisions that considers the degree of optimism of the decision maker. It is defined as follows.

Definition 1. An OWA operator of dimension n is a mapping OWA: $R^n \to R$ that has an associated weighting vector W of dimension n with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$, such that:

$$OWA(a_1, a_2, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \quad (1)$$

where b_j is the jth largest of the a_i.

Note that different properties can be studied such as the distinction between descending and ascending orders, different measures for characterizing the weighting vector and different families of OWA operators. Note that it is commutative, monotonic, bounded and idempotent. For further reading, refer, for example to [30–35].