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Soft discernibility matrix and its applications in decision making

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ABSTRACT

In this paper, we aim to solve the problems of decision making by introducing soft discernibility matrix in soft sets. Firstly, the notion of soft discernibility matrix is introduced in soft sets, and some properties associated with the soft discernibility matrix are researched. Secondly, an novel algorithm based on soft discernibility matrix is proposed to solve the problems of decision making. It can find not only the optimal choice object, but also an order relation of all the objects easily by scanning the soft discernibility matrix at most one time, rather than calculating the choice value. Finally, the weighted soft discernibility matrix is introduced in soft sets and its application to decision making is also investigated.

1. Introduction

We all know that there are full of uncertainty, vagueness and imprecision in real world. That is, most of the notions we are meeting in everyday life are vague rather than precise. Classical mathematical tools may not be successfully used to solve these problems, because classical mathematics requires that all mathematical concepts must be exact, otherwise precise reasoning would be impossible. In recent years, engineers and researchers have become interested in model vagueness, because many practical problems arise in such areas as economics, environmental science, engineering, medical science and social science involve data containing a wide range of uncertainties.

Various types of theories such as rough set theory [20], probability theory, fuzzy set theory [22] and interval mathematics [23] are effective mathematical approaches to model vagueness, but each of these theories has its inherent limitations, which are pointed out in [12]. In 1999, soft set theory [12] was firstly proposed by Molodtsov. It is considered as a new mathematical tool for dealing with uncertainties which is free from the inadequacy of parameter tools. In soft set theory, the problem of setting the membership function simply does not arise as in fuzzy set theory, which makes the theory convenient and easy to use in practice. Soft set theory has been proven useful in many different fields such as the smoothness of functions, probability theory, measurement theory, and operations research [12,13].

In recent years, research on soft sets has been very active and many important results have been achieved in the theoretical aspect [2–4,11–14,16,19,26]. At the same time, there also have some progress concerning practical applications of soft set theory, especially the use of soft sets in decision making [1,5–10,17,18].

Applications of soft sets in decision making problems have been studied by many authors in different contexts [1,5–10,17,18,27]. In [5], the authors defined soft matrix in soft sets and researched operations of soft matrix. Then a new method based on soft matrix was proposed to solve the problems of decision making. In [6], the authors applied the properties of operations of soft sets to solve the decision making problems. Maji et al. [8] first applied soft sets to solve decision making problems that is based on the concept of knowledge reduction. Chen et al. [7] presented a parameter reduction of soft sets and its applications. They pointed out that the results of reduction proposed by Maji et al. are incorrect, and the algorithm which first to compute the parameter reduction and then to compute the choice value to select the optimal objects is unreasonable. They also pointed out that the idea of attributes reduction in rough sets is quite different from the parameter reduction in soft sets. However, due to the method in [7] only based on the optimal choice object related to each object for soft set reduction, the problems of the sub-optimal choice object is not considered. Kong et al. [9] pointed out some odd situations which may occur when the method of parameter reduction in [7] is applied. Then, they proposed a method of normal parameter reduction in soft sets. With this technique, the optimal and suboptimal choice objects are still preserved. In [27], the authors proposed some improved algorithms which require relatively fewer calculations, and they also pointed out the choice value based approach and comparison score based

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approach are equivalent for crisp soft set based decision making problems.

Ali [1] proposed a concept of soft equivalence relation, which may act as a bridge between soft sets and rough sets. He also gave a method of parameters reduction in soft sets, which is very much similar to knowledge reduction in rough sets. So we can solve the problems in soft sets with the help of rough sets.

All approaches to solve decision making problems in soft sets mentioned above mainly to calculate the parameters reduction firstly, then select the optimal choice objects. And the main idea of parameters reduction is to reduce the number of parameters which have no influence to keep the optimal choice objects unchanged in soft sets.

In this paper, we aim to develop a method of applying soft discernibility matrix to solve the decision making problems. The notion of soft discernibility matrix is introduced in soft sets firstly. And then a new method is proposed to solve the problems of decision making. This method can find not only the optimal choice object, but also an order relation of all objects easily by scanning the soft discernibility matrix at most one time.

The rest of the paper is organized as follows. In Section 2, some basic concepts about information systems and soft set theory are reviewed. In Section 3, the notion of soft discernibility matrix is introduced in soft sets, and some properties associated with the soft discernibility matrix are researched. At last, a new technique based on the soft discernibility matrix is proposed to solve the problems of decision making. Section 4 is devoted to weighted soft discernibility matrix with applications to decision making problems. Finally, conclusions and the future works are given in Section 5.

2. Preliminaries

2.1. Information system

In this section some basic notions of information systems are reviewed, which will be required in later sections. Let U be a nonempty finite set, E also a nonempty finite set and $A \subseteq E$.

Definition 1. ([21]) The quadruple (U, A, V, f) is called an information system, or an information table, a database system, where $U = \{x_1, x_2, \ldots, x_n\}$ is a universe containing all interested objects, every $x_i \in U (i \leq n)$ is called an object, $A = \{a_1, a_2, \ldots, a_m\}$ is a set of attributes, each $a_j \in A (j \leq m)$ is called an attribute, $V = \bigcup_{j=1}^{m} V_j$, where V_j is the value set of the attribute a_j and $F = \{f_1, f_2, \ldots, f_m\}$, where $f_j : U \to V_j$.

Definition 2. ([21]) R is an equivalence relation over U, the partition of U determined by indiscernibility relation R is denoted by

$$\mathcal{A} = U/R = \{[x_i]_R : x_i \in U\}$$

where $[x_i]_R = \{x_j : (x_i, x_j) \in R\}$ is called an equivalence class containing x_i.

Although rough sets and soft sets are two different mathematical tools for dealing with the model of vagueness, there are some interesting connections between them. We notice that information systems and soft sets are closely related.

2.2. Soft sets

In this section we will review some basic concepts and properties about soft sets. Let U be a nonempty initial set of objects, E be a set of parameters in relation to objects in U, $P(U)$ be the power set of U, and $A \subseteq E$. The definition of soft sets is given as following.

Definition 3. ([12]) let U be a nonempty finite set and $U = \{h_1, h_2, \ldots, h_m\}$, E be a set of parameters and $E = \{e_1, e_2, \ldots, e_n\}$. A pair (F, A) is called a soft set over U, if F is a function from a subset A of E to the power set of U. That is, a soft set is a parameterized family of subsets of U.

Every soft set over U can be considered as a parameterized family of subsets of the universe U. For all $e \in A$, each set $F(e)$ may be treated as the set of e-elements of the soft set (F, A), or as the set of e-approximate elements of the soft sets.

Proposition 1. *([25]) Each soft set may be considered as an information system.*

If (F, E) be a soft set over the universe U', $S = (U, A, V, f)$ be an information system. Obviously, the universe U' in (F, E) may be considered as the universe U, the parameters set E may be considered as the attributes set A. The information function f is defined by

$$f(x) = \begin{cases} 1, x \in F(e) \\ 0, x \notin F(e) \end{cases}$$

that is, when $x_i \in F(e_j)$, where $x_i \in U$ and $e_j \in E$, then $f(x_i, e_j) = 1$, otherwise $f(x_i, e_j) = 0$. So we have $V = \{0, 1\}$. Therefore, a soft set (F, E) may be considered as an information system $S = (U, A, V_{\{0,1\}}, f)$.

As we have known that each soft set can be represented in the form of an information system, at the same time, each information system can also be converted into a soft set. Let $S = (U, A, V, f)$ be an information system, $B = \bigcup_{a \in A} \{a\} \times V_a$ be a set of parameters, and then a soft set (F, B) can be defined by setting $F(a, v) = \{x \in U : a(x) = v\}$, where $v \in V_a$ for all $a \in A$.

Definition 4. ([2]) If $F : A \to P(U \times U)$ be a mapping from a subset of parameter A to the power set of $U \times U$, then the soft set (F, A) over $U \times U$ is called a soft binary relation over U.

Definition 5. ([1]) A soft binary relation (F, A) over a set U is called a soft equivalence relation over U, if $F(\alpha) \neq \emptyset$ is an equivalence relation on U for all $\alpha \in A$.

It is well known that there exists a one-to-one relationship between the equivalence relation and partition. Therefore, a soft equivalence relation over U provides us a parameterized collection of partitions of U.

Definition 6. ([1]) Suppose (F, A) is a soft equivalence relation over U, for each equivalence relation $F(\alpha)$, the notion of equivalence class over it is defined as $[x]_{F(\alpha)} = \{y : (x, y) \in F(\alpha), y \in U\}$.

We also observe that there is an indiscernibility relation induced by the soft set (F, A) itself. This indiscernibility relation is obtained by intersection of all the equivalence relations defined by parameters. Let us say

$$IND(F, A) = \bigcap_{e_i \in A} F(e_i)$$

Suppose (F, A) is a soft set over U, where $U = \{h_1, h_2, \ldots, h_m\}$. The partition of U determined by the indiscernibility relation $IND(F, A)$ can be denoted by $U|IND(F, A)$, and

$$U|IND(F, A) = \{C_1, C_2, \ldots, C_i\} \quad (i \leq m)$$

where $C_i = \{[h_j]_{IND(F,A)} : h_j \in U\}$.

In [1], the concept of decision parameter $D = \sum h_{ij}$ was introduced, where h_{ij} is the value for the ith object h_i corresponding to the jth parameter e_j. However, it is well known that there is a straightforward relationship between the conditional parameters and the decision parameter in soft sets. Therefore, it is impossible to keep the optimal choice object unchanged by using the method of attribute reduction in rough sets. Now some basic concepts associated with the consistent decision table in soft sets, which are similar with those in decision table in rough sets will be revised.