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A learning-guided multi-objective evolutionary algorithm for constrained portfolio optimization

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ABSTRACT

Portfolio optimization involves the optimal assignment of limited capital to different available financial assets to achieve a reasonable trade-off between profit and risk objectives. In this paper, we studied the extended Markowitz's mean-variance portfolio optimization model. We considered the cardinality, quantity, pre-assignment and round lot constraints in the extended model. These four real-world constraints limit the number of assets in a portfolio, restrict the minimum and maximum proportions of assets held in the portfolio, require some specific assets to be included in the portfolio and require to invest the assets in units of a certain size respectively. An efficient learning-guided hybrid multi-objective evolutionary algorithm is proposed to solve the constrained portfolio optimization problem in the extended mean-variance framework. A learning-guided solution generation strategy is incorporated into the multi-objective optimization process to promote the efficient convergence by guiding the evolutionary search towards the promising regions of the search space. The proposed algorithm is compared against four existing state-of-the-art multi-objective evolutionary algorithms, namely Non-dominated Sorting Genetic Algorithm (NSGA-II), Strength Pareto Evolutionary Algorithm (SPEA-2), Pareto Envelope-based Selection Algorithm (PESA-II) and Pareto Archived Evolution Strategy (PAES). Computational results are reported for publicly available OR-library datasets from seven market indices involving up to 1318 assets. Experimental results on the constrained portfolio optimization problem demonstrate that the proposed algorithm significantly outperforms the four well-known multi-objective evolutionary algorithms with respect to the quality of obtained efficient frontier in the conducted experiments.

1. Introduction

Portfolio selection problem is a well-studied topic in finance and it is concerned with the optimal allocation of a limited capital among a finite number of available risky assets, such as stocks, bonds, and derivatives in order to gain the possible highest future wealth. Markowitz's mean-variance model [40,41] is considered to play an important role in the development of Modern Portfolio Theory. The mean-variance (MV) model assumes that the future market of the assets can be correctly reflected by the historical market of the assets. It considers the trade-off between risk and reward in selecting *efficient* portfolios. A portfolio is considered to be *efficient* if it provides the highest possible reward for a given risk or alternatively, if it presents the least possible risk for a given level of profit. The reward (profit) of the portfolio is measured by the average expected return of those individual assets in the portfolio whereas the risk is measured by its combined total variance.

While investing the capital within the MV framework, investors have *two* objectives: maximizing the total profit and minimizing the total risk of their portfolios. With these two conflicting objectives to be optimized simultaneously, the portfolio selection problem can be classified as a multi-objective optimization problem. A single solution that optimizes all the conflicting objectives simultaneously hardly exists in practice. Instead, there exists a set of acceptable 'compromise' solutions which are optimal in such a way that no other solutions are superior to them when all objectives are considered simultaneously. Such solutions are referred to as *efficient* solutions, *non-dominated* solutions or *Pareto-optimal* solutions.

The collection of such efficient portfolios conveying the compromise between risk and return is called the *efficient frontier* or *Pareto-optimal front*. The efficient frontier helps investors to visualize the risk and return trade-off curve in a two-dimensional graph with risk on the horizontal axis and expected return on the vertical axis (see Fig. 13).

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Since the Markowitz's pioneering work, many researchers have pursued studies for efficient algorithms [27,29,43,52] to compute the efficient frontier of the MV model. However, the classic MV model assumes a perfect market where short sales are disallowed, securities can be traded in any (non-negative) fractions, no limitation on the number of assets in the portfolio, investors have no preferences over assets and they do not care about different assets types in their portfolios. In practice, these assumptions are unrealistic. As a result, several extensions and modifications have been proposed to address the real-world constraints. In this paper, we extended the basic MV model to include four practical constraints as follows:

Cardinality constraint

Cardinality constraint limits the number of assets (K) that compose the portfolio. Very often in practice, investors prefer to have a limited number of assets included in their portfolio since the management of many assets in the portfolio is tedious and hard to monitor. They also intend to reduce transaction costs and/or to assure a certain degree of diversification by limiting the maximum number of assets in their portfolios.

Floor and ceiling constraints

The floor and ceiling constraints specify the minimum and maximum limits on the proportion of each asset that can be held in a portfolio. In practice, investors prefer to avoid excessive administrative costs for very small holdings of assets in the portfolio and/or some institutional policies require to model their policies on the lower and upper bounds of each asset in the portfolio. The floor and ceiling constraint is also known as bounding or quantity constraints.

Pre-assignment constraint

The pre-assignment constraint is usually used to model the investor's subjective preferences. An investor may intuitively wish some specific assets to be included in the portfolio, with its proportion fixed or to be determined.

Round lot constraint

Round lot constraint requires the number of any asset in the portfolio to be in exact multiple of the normal trading lots. In practice, several market securities are traded as multiples of minimum lots.

These four constraints stated above are *hard* in the sense that they have to be satisfied at any time. In practice, portfolios are composed of markets with hundreds to thousands of available assets, and the calculation of risk measures grows quickly in relation to the number of assets. By introducing the cardinality constraint alone already transforms the classic quadratic optimization model into a mixed-integer quadratic programming problem which is an NP-hard problem [6,47]. There are several exact approaches proposed in the literature for cardinality constrained portfolio optimization problem [5,6,35,47]. However, all these works relaxed the cardinality constraint as an inequality constraint allowing the number of assets in the portfolio to vary with maximum bound (K) and the results showed that they are able to handle the test problems with limited size (up to 500 assets). On the other hand, Gulpinar et al. [26] considered the strict cardinality constraint and computational results are performed on a small test problem involving 98 assets.

When additional constraints are added to the basic MV model, the problem thus becomes more complex and the exact optimization approaches run into difficulties to deliver solutions within reasonable time for large problem size. As a result, this motivates the investigation of approximate algorithms such as meta-heuristics [33] and hybrid meta-heuristics [56,45]. In general, meta-heuristics cannot guarantee the optimality of the solution, but they are efficient in finding the optimal or near optimal solutions in a reasonable amount of time.

There exist many studies which applied meta-heuristics or other techniques to solve portfolio optimization problem [21,39]. The recent research in portfolio optimization problem is widely carried out by incorporation of constraints in the problem model and/or handling the problem as a multi-objective one. Although the portfolio optimization problem involves two conflicting objectives, many studies in the literature [11,17,20,37] have been performed as single objective meta-heuristics approaches with aggregating function that combines two objectives into a single scale objective, and in which the weights are varied to generate the set of efficient solutions for portfolio selection problems with cardinality and quantity constraints. Mansini and Speranza [38] showed that the portfolio selection problem with round lot constraint is an NP-complete problem and proposed three mixed integer linear programming heuristic algorithms to solve the problem. Lin and Liu [36] proposed a genetic algorithm with three different models for portfolio selection problems with round lots. Chang et al. [11] and Gaspero et al. [25] discussed the pre-assignment briefly but had not addressed the constraint in their experiments.

In recent years, many publications had discussed the portfolio optimization problems with multi-objective evolutionary algorithms by considering a subset of the real-world constraints. Diosan [22] and Mishra et al. [42] applied several well-known multi-objective evolutionary algorithms to solve the unconstrained portfolio optimization problem. Recently, Krink et al. [34] also proposed an algorithm called DEMPO inspired by the NSGA-II algorithm [19]. The difference between NSGA-II and DEMPO is that Differential Evolution (DE) is used instead of Genetic Algorithm (GA) to generate new candidate solutions during the evolution. DEMPO is applied to solve the basic portfolio optimization problem based on Value-at-Risk risk measure and experimental results show that DEMPO outperforms NSGA-II. Armananzas and Lozano [3] studied greedy search, simulated annealing (SA) and ant colony optimization (ACO) algorithms in a multi-objective framework to solve the portfolio selection problem with cardinality constraints.

Anagnostopoulos and Mamanis [2] considered the extended MV model with cardinality and quantity constraints and tested five advanced MOEAs to investigate the performance. The cardinality constraint considered in their work is relaxed and as a result a portfolio can be composed of any number of assets with maximum bound (K). The experimental results confirmed that all multi-objective algorithms considered outperformed the single objective evolutionary algorithm. The results also concluded that SPEA-II [60] performed the best among those algorithms tested. Branke et al. [7] also presented an envelope-based MOEA integrating the NSGA-II [18] and the critical line algorithm. Chaim et al. [12] proposed an order-based solution representation and considered the cardinality constraint as a soft constraint and quantity constraint as a hard constraint. In their work, the cardinality constraint was relaxed and hence it was allowed to vary the number of assets in the portfolio from the minimum limit to the maximum limit.

Streichert et al. [55,54] applied a multi-objective evolutionary algorithm (MOEA) to solve the portfolio selection problems with cardinality, floor and round lot constraints. These works studied various crossover operators adopting hybrid chromosome representation with binary and real values. This hybrid encoding enhances the performance of the algorithm significantly regardless of the choice of crossover operators. Skolpadungket et al. [50] also studied the portfolio selection problems with cardinality, floor and round lot constraints and tested them with various MOEAs. They adopted the same hybrid encoding as Streichert et al. [55,54].