Applied Soft Computing xxx (2014) xxx–xxx

Contents lists available at ScienceDirect

Applied Soft Computing

journal homepage: www.elsevier.com/locate/asoc

The generalized covering traveling salesman problem

Q1 Mohammad H. Shaelaie [a], Majid Salari [a,*], Zahra Naji-Azimi [b]

[a] *Department of Industrial Engineering, Ferdowsi University of Mashhad, Mashhad, Iran*
[b] *Department of Management, Ferdowsi University of Mashhad, Mashhad, Iran*

ARTICLE INFO

Article history:
Received 20 March 2013
Received in revised form 25 August 2014
Accepted 25 August 2014
Available online xxx

Keywords:
Traveling salesman problem
Humanitarian relief transportation
Memetic algorithm
Variable neighborhood search

ABSTRACT

In this paper we develop a problem with potential applications in humanitarian relief transportation and telecommunication networks. Given a set of vertices including the depot, facility and customer vertices, the goal is to construct a minimum length cycle over a subset of facilities while covering a given number of customers. Essentially, a customer is covered when it is located within a pre-specified distance of a visited facility on the tour. We propose two node-based and flow-based mathematical models and two metaheuristic algorithms including memetic algorithm and a variable neighborhood search for the problem. Computational tests on a set of randomly generated instances and on set of benchmark data indicate the effectiveness of the proposed algorithms.

1. Introduction

The traveling salesman problem (TSP) is one of the most studied problems in the field of combinatorial optimization. Given a graph $G = (N, E)$, the TSP is to find a minimum length Hamiltonian cycle of the vertices in N in which each vertex has to be visited exactly once [13].

The quota traveling salesman problem (QTSP) is a generalization of the TSP in which the salesman has to visit a given quota of vertices while minimizing the total travel cost [4,10]. In this problem, each vertex has a pre-determined amount of prize that could be collected by the salesperson by being visited on the tour. The k-TSP is a special case of the QTSP in which each vertex has one unit of prize and the goal is to visit k vertices, in order to collect k units of prize [15,30].

Introduced by Balas in 1989 [5], in the prize collecting traveling salesman problem (PCTSP) it is given a set of vertices in which each vertex has a pre-determined amount of prize. The goal of the PCTSP is to collect a certain amount of prize, while minimizing the routing and penalty costs. Essentially, the routing cost is the total distance traveled by the salesman and the penalty cost is the cost occurred by not visiting a vertex on the tour [5,9].

Many variations of the standard TSP are introduced in the literature including the online TSP [38], the TSP with pickup-and-delivery [42], clustered TSP [6], generalized TSP [27,29], multi-depot multiple TSP [17] and etc. To the complete survey on the TSP and its variants, we refer the interested readers to the book by Gutin and Punnen [19].

There is a vast body of literature dedicated to covering problems. The purpose of such problems is to satisfy the customers' demand in two different ways. Essentially, different customers' demand can be provided at the same location at which they are located or it can be delivered at a destination within a pre-specified distance of the customer locations [41].

Several variations of the TSP with potential applications in route designing of the healthcare teams in developing countries [16], and applications from design of the distributed networks [21] have been introduced. In these two problems and some others which are proposed in the literature, there is no need to visit all the customers on the tour structure and the objectives can be satisfied by visiting a limited number of customers. As an example we can refer to the case of emergency management in which it is not possible for the medical team to visit all the customers on the tour. In such situation, a subset of customers can be visited on the tour and the rest of them are covered, i.e., located within a pre-specified distance of at least on visited customer.

The covering salesman problem (CSP) is a generalization of the TSP in which we have to satisfy all the customers' demand by visiting or covering them [11]. Essentially, for each customer, say i, it is given a covering radius d_i, within that all the located customers will be covered. The goal of the CSP is to construct a minimum length tour over a subset of the given customers such that each customer, not visited on the tour, is within the covering distance of at least one

* Corresponding author. Tel.: +98 511 8805113.
E-mail addresses: hesam_shaelaie@stu.um.ac.ir (M.H. Shaelaie), msalari@um.ac.ir, salarimajid@yahoo.com (M. Salari), znajiazimi@um.ac.ir (Z. Naji-Azimi).

http://dx.doi.org/10.1016/j.asoc.2014.08.057
1568-4946/© 2014 Published by Elsevier B.V.

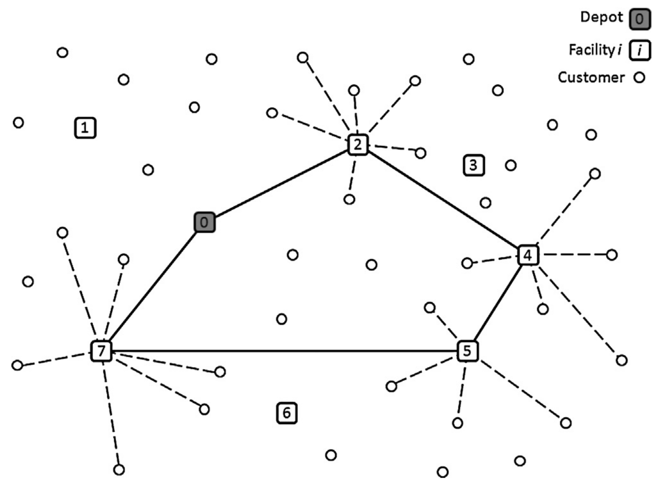

Fig. 1. An illustrative example of the problem.

visited customer [11]. Many papers have discussed on the application of this problem and some of its generalization in the fields of emergency management and disaster planning [1,34,35,37].

What happens in reality when satisfying a given set of customers' demand, is somehow different from the assumptions made in many problems, introduced in the literature. As an example, usually the distribution centers cannot be constructed at the same location at which the customer zones are located. Essentially, there can be a situation in which it is not possible for the health care team to assess the customer zone by vehicles. So, they need to locate the facilities in a location different from the customer zone. Moreover, in the QTSP case the customers can travel to a facility location to receive their demands.

Taking into account the above assumptions, in this paper we propose a generalization of the TSP with applications in humanitarian relief transportation. Suppose we are given a set of vertices including a central depot, facility and customer vertices. Each customer i has one unit of demand (prize) that will be satisfied when it is located within a certain coverage radius of at least one facility j visited by the tour. Essentially, each facility j has a coverage radius d_j within that all the available customers are covered. The goal of the problem is to construct a minimum length Hamiltonian cycle over a subset of facilities, while collecting the specified amount of prize P.

The introduced problem is a generalization of the CSP, TSP, k-TSP and the QTSP and all applications already introduced for these problems could be easily extended to the new developed problem. Letting $P=21$, Fig. 1 represents an example in which 4 facilities have been visited by the tour and 21 customer vertices are covered by the visited facilities.

The paper is organized as follows. Section 2 develops a formal description of the problem. The details of the proposed solution methods are provided in Section 3. Extensive computational tests for the studied problem and one of its variants (CSP) are reported in Section 4, followed by conclusion and future directions given in Section 5.

2. Problem modeling

The problem is defined on a directed graph $G=(V, A)$, where $V=\{0\}\cup W\cup T$ contains three sets of vertices including the depot, $\{0\}$, the set of customers, W, and the set of facilities, T. Moreover, the set of arcs is given by $A=\{(i,j)|i,j\in T\cup\{0\}\}$. To each $(i,j)\in A$ is associated a travel cost represented by t_{ij} and each customer i has a prize, p_i, that will be covered by the tour when it is located within the predefined coverage distance of at least one routed facility. We denote by T_i those facilities that are able to cover customer i. The goal of the problem is to collect a pre-determined amount of prize, P, while minimizing the total traveled length.

The decision variables to model the problem are defined as follows:

$$x_{ij}=\begin{cases}1 & \text{if arc } (i,j) \text{ is visited by the tour,}\\ 0 & \text{otherwise.}\end{cases}\quad \forall(i,j)\in A \tag{1}$$

$$z_{ij}=\begin{cases}1 & \text{if the demand of customer i is allocated to the facility j,}\\ 0 & \text{otherwise.}\end{cases}\quad \forall i\in W, j\in T \tag{2}$$

Finally, for each $i\in T$, u_i is a continuous variable representing the tour load before visiting vertex i.

In what follows, we extend two mathematical models for the studied problem. Based on the formulations developed for the TSP and its variants, we divide the models into two groups namely, node-based and flow-based formulations.

2.1. Node-based formulation

In this section, we develop a node-based formulation for the studied problem. This kind of formulation, applies the well known Miller–Tucker–Zemlin subtour elimination constraints proposed for the TSP [14,25,31].

$$\min \sum_{(i,j)\in A} t_{ij}x_{ij} \tag{3}$$

subject to:

$$\sum_{i\in W}\sum_{j\in T_i} p_i z_{ij} \geq P \tag{4}$$

$$\sum_{j\in T_i} z_{ij} \leq 1 \quad \forall i\in W, \tag{5}$$

$$\sum_{j\in T\cup\{0\}} x_{0j} = 1 \tag{6}$$

$$\sum_{j\in T\cup\{0\}} x_{ij} = \sum_{j\in T\cup\{0\}} x_{ji} \quad \forall i\in T\cup\{0\}, \tag{7}$$

$$z_{ij} \leq \sum_{k\in T\cup\{0\}} x_{kj} + \sum_{k\in T\cup\{0\}} x_{jk} \quad \forall i\in W, j\in T_i, \tag{8}$$

$$u_i - u_j + (|T|+1)X_{ij} \leq |T| \quad \forall i,j\in T, \tag{9}$$

$$x_{ij}\in\{0,1\} \quad \forall i,j\in T\cup\{0\}, \tag{10}$$

$$z_{ij}\in\{0,1\} \quad \forall i\in W, j\in T_i, \tag{11}$$

$$u_j \geq 0 \quad \forall j\in T\cup\{0\}. \tag{12}$$

The objective function (3) is to minimize the total traveling cost. Constraint (4) assures the total collected prize, by the eligible facilities, to be greater than or equal to the pre-specified value P. For each $i\in W$, constraint (5) assures customer i to be allocated to at most one facility $j\in T_i$. Constraint (6) makes sure that the tour to be started from the depot, followed by the in-degree and out-degree constraints (for each $i\in T\cup\{0\}$) represented by constraints set (7). For each customer i and facility $j\in T_i$, constraint (8) shows that, i could be allocated to j only if j is visited by the tour. Constraint set (9) models the sub-tour elimination constraints. Finally, constraints (10)–(12) define the model variables.

Please cite this article in press as: M.H. Shaelaie, et al., The generalized covering traveling salesman problem, Appl. Soft Comput. J. (2014), http://dx.doi.org/10.1016/j.asoc.2014.08.057