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Optimal control for stochastic linear quadratic singular neuro Takagi–Sugeno fuzzy system with singular cost using genetic programming

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ABSTRACT

In this paper, optimal control for stochastic linear quadratic singular neuro Takagi–Sugeno (T-S) fuzzy system with singular cost is obtained using genetic programming (GP). To obtain the optimal control, the solution of matrix Riccati differential equation (MRDE) is computed by solving differential algebraic equation (DAE) using a novel and nontraditional GP approach. The obtained solution in this method is equivalent or very close to the exact solution of the problem. Accuracy of the solution computed by GP approach to the problem is qualitatively better. The solution of this novel method is compared with the traditional Runge–Kutta (RK) method. A numerical example is presented to illustrate the proposed method.

1. Introduction

A fuzzy system consists of linguistic IF-THEN rules that have fuzzy antecedent and consequent parts. It is a static nonlinear mapping from the input space to the output space. The inputs and outputs are crisp real numbers and not fuzzy sets. The fuzzification block converts the crisp inputs to fuzzy sets and then the inference mechanism uses the fuzzy rules to produce fuzzy conclusions or fuzzy aggregations and finally the defuzzification block converts these fuzzy conclusions into crisp outputs. The fuzzy system with singleton fuzzifier, product inference engine, center average defuzzifier and Gaussian membership functions is called a standard fuzzy system [1].

Two main advantages of fuzzy systems for the control and modeling applications are (i) fuzzy systems are useful for uncertain or approximate reasoning, especially for the system with a mathematical model that is difficult to derive and (ii) fuzzy logic allows decision making with the estimated values under incomplete or uncertain information [2].

Neural networks or simply neural nets are computing systems, which can be trained to learn a complex relationship between two or many variables or data sets. Having the structures similar to their biological counterparts, neural networks (NNs) are representational and computational models. NNs are processing information in a parallel distributed fashion composed of interconnecting simple processing nodes [3]. Neural net techniques have been successfully applied in various fields such as function approximation, signal processing and adaptive (or) learning control for nonlinear systems. Using neural networks, a variety of off line learning control algorithms have been developed for nonlinear systems [4,5].

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Neuro fuzzy systems are a combination of two popular soft computing techniques: neural networks and fuzzy systems. Also NNs have the capability to learn from examples, yet the learned knowledge can not be represented explicitly. On the other hand, knowledge in fuzzy systems is represented via explicit fuzzy if then rules, yet fuzzy systems have no learning capability. Neuro fuzzy system is a hybrid approach in which a fuzzy system is trained using techniques similar to those applied to neural networks. One of the first neuro fuzzy systems was the adaptive network-based fuzzy inference system (ANFIS) [6]. ANFIS represents a Takagi–Sugeno fuzzy system as a multilayer feedforward network which can be trained via backpropagation algorithm. Neural networks and fuzzy systems can be combined to join its advantages and to cure its individual illness. Neural networks introduce its computational characteristics of learning in the fuzzy systems and receive from them the interpretation and clarity of systems representation. Thus, the disadvantages of the fuzzy systems are compensated by the capacities of the neural networks. Hayashi et al. [7] showed that a feedforward neural network could approximate any fuzzy rule based system and any feedforward neural network may be approximated by a rule based fuzzy inference system [8]. Fusion of artificial neural networks and fuzzy inference systems have attracted the growing interest of researchers in various scientific and engineering areas due to the growing need of adaptive intelligent systems to solve the real world problems [9–11].

Genetic programming is an evolutionary algorithm that attempts to evolve solution to the given problem by using concepts taken from naturally occurring evolving process. The technique is based on the evolution of a large number of candidate solutions through genetic operations such as reproduction, crossover and mutation. It is based upon the genetic algorithm(GA) [12], which exploits the process of natural selection based on a fitness measure to breed a population of trial solutions that improve over time. While GA usually operates on (coded) strings of numbers, GP uses the principles and ideas from biological evolution to guide the computer to acquire desired solution. The search space is too large to attempt a brute force search, the method must be utilized to reduce the number of examined solutions. In this search, initially the population looks a bit like a cloud of randomly selected points, but that generation after generation it moves in the search space following a well defined trajectory. The generation is achieved with the help of grammatical evolution, because grammatical evolution can produce programmes in an arbitrary language. The genetic operations are faster and also because it is more convenient to symbolically differentiate mathematical expression. The code production is performed using a mapping process governed by grammar expressed in Backus Naur Form(BNF) [13]. In analogy to nature, the potential solution is an individual in some collection or population of potential solutions. The individuals who are stronger, meaning higher ranked according to fitness function, will be used to determine the next collection of potential solution. A new generation will be arisen by employing analogs of reproduction, crossover and mutation.

This means that GP has advantages over other algorithms as it can perform optimization at a structural level. This enabled Koza [14] to demonstrate the application of GP algorithm to a number of problem domains, including regression, control and classification. Research in this area has grown rapidly and encompassed a wide range of problems. GP techniques have been successfully applied in various engineering fields like signal processing [15], electrical circuit design [16], scheduling [17], process controller evolution [18] and modelling of both steady-state and dynamic processes [19].

In this paper, optimal control of a stochastic linear quadratic singular neuro T-S fuzzy system is obtained using genetic programming. The linear T-S fuzzy system is the most popular fuzzy model due to its intrinsic analysis: the linear matrix inequality(LMI)-based fuzzy controller is to minimize the upper bound of the performance index; structure oriented and switching fuzzy controllers are developed for more complicated systems [20]; the optimal fuzzy control technique is used to minimize the performance index from local-concept or global-concept approaches [21,22].

Stochastic linear quadratic regulator (LQR) problems have been studied by many researchers [23,24]. Chen et al. [25] have shown that the stochastic LQR problem is well posed if there are solutions to the Riccati equation and then an optimal feedback control can be obtained. For LQR problems, it is natural to study an associated Riccati equation. However, the existence and uniqueness of the solution of the Riccati equation in general, seem to be very difficult problems due to the presence of the complicated nonlinear term. Zhu and Li [26] used the iterative method for solving stochastic Riccati equations for stochastic LQR problems. There are several numerical methods to solve conventional Riccati equation. As a result of the nonlinear process, essential error accumulations may occur. Recently the conventional Riccati equation has been analyzed using neural network approach and genetic programming approach see [27–34] to minimize the error. A variety of numerical algorithms [35] have been developed for solving the algebraic Riccati equation.

Singular systems contain a mixture of algebraic and differential equations. In that sense, the algebraic equations represent the constraints to the solution of the differential part. These systems are also known as degenerate, differential algebraic, descriptor or semi state and generalized state space systems. The complex nature of singular system causes many difficulties in the analytical and numerical treatment of such systems, particularly when there is a need for their control. The system arises naturally as a linear approximation of system models or linear system models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large scale systems, robotics, biology, etc., see [36,37]. As the theory of optimal control of linear systems with quadratic performance criteria is well developed, the results are most complete and close to use in many practical designing problems. The theory of the quadratic cost control problem has been treated as a more interesting problem. The optimal feedback with minimum cost control has been characterized by the solution of a Riccati equation. Da Prato and Ichikawa [38] showed that the optimal feedback control and the minimum cost are characterized by the solution of a Riccati equation. Solving the Matrix Riccati differential equation (MRDE) is the central issue in optimal control theory.

Although parallel algorithms can compute the solutions faster than sequential algorithms, there have been no report on genetic programming solutions for MRDE [34]. This paper focuses upon the implementation of genetic programming approach for solving MRDE in order to get the optimal solution.

This paper is organized as follows. In Section 2, the statement of the problem is given. In Section 3 and 4, solution of the MRDE is presented. In Section 5, numerical example is discussed to illustrate the proposed method. The final conclusion section demonstrates the efficiency of the method.

2. Statement of the problem

In this section, a class of adaptive networks is proposed. The proposed architecture is referred to as ANFIS. For simplicity, the fuzzy inference system has two inputs x_1 and x_2 and one output $z = f(x_1, x_2)$. Suppose that the rule base contains two fuzzy if-then rules of Takagi and Sugeno's type [39]:

$$\begin{aligned} &Rule1: If \quad x_1 \quad is \quad A_1 \quad and \quad x_2 \quad is \quad B_1, then \quad f_1 = p_1x_1 + q_1x_2 + r_1, \\ &Rule2: If \quad x_1 \quad is \quad A_2 \quad and \quad x_2 \quad is \quad B_2, then \quad f_2 = p_2x_1 + q_2x_2 + r_2. \end{aligned} \tag{187}$$