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Non-similar Blasius and Sakiadis flow of a non-Newtonian Carreau fluid

Asterios Pantokratoras*

School of Engineering, Democritus University of Thrace, 67100 Xanthi, Greece

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ABSTRACT

The Blasius and Sakiadis flow of a non-Newtonian Carreau fluid is considered in the present paper. The boundary layer equations are transformed into non-dimensional form and a new dimensionless parameter (Deborah number) is introduced. The transformed boundary layer equations are solved with the finite difference method. The problem is non-similar and is governed by the Deborah number, the power-law index and the non-dimensional distance along the plate. Velocity profiles and wall shear stress have been calculated for both cases and a comparison is made between Blasius and Sakiadis flow.

1. Introduction

The problem of fluid flow along a stationary, horizontal, surface situated in a fluid stream moving with constant velocity is a classical problem of fluid mechanics that has been solved for the first time in 1908 by Blasius [1]. In the above problem the fluid motion is produced by the free stream. A similar problem occurs when an infinite surface moves with a constant velocity in a calm fluid. This problem has been treated for the first time by Sakiadis [2]. The above flows have been studied for usual Newtonian fluids as is water and air.

Many rheological models had been tried until 1972 to describe adequately the behavior of non-Newtonian viscoelastic materials. Carreau [3] introduced a successful model which is used extensively up to date. Since then the Carreau model has been used to simulate non-Newtonian flows around spheres [4–6], over cylinders [7–9], in cavities [10,11], in pipes [12], in channels [13–16] and in intestines [17] to mention just a few of them. After an intensive investigation made in the literature no simulation has been found concerning the flow of a Carreau fluid over a stationary (Blasius) or moving surface (Sakiadis) flow which are two fundamental flows in fluid mechanics and this is the target of the present work.

It is reminded here that many non-Newtonian fluids are used in chemical engineering (polymer liquids, silicone oils). An example is given in the following table which contains the properties of some silicone oils (Table 1).

2. The mathematical model

Consider the flow of a non-Newtonian Carreau fluid along a horizontal surface with u and v denoting respectively the velocity components in the x and y directions, where x is the coordinate along the surface and y is the coordinate perpendicular to x. The constitutive equation for a Carreau fluid is given as [8]

$$\tau = \mu_\infty + (\mu_0 - \mu_\infty)\left[1 + (\lambda\dot{\gamma})^2\right]^{(n-1)/2}\dot{\gamma} \tag{1}$$

where μ_0 and μ_∞ are the viscosities corresponding to zero and infinite shear-rate, λ is the time constant, $\dot{\gamma}$ is strain rate tensor and n is the power-law index. The fluid is characterized as shear-thinning for $0 < n < 1$, shear-thickening for $n > 1$ and Newtonian for $n = 1$. At low shear rate the Carreau fluid behaves as Newtonian fluid and at high shear rate as power-law fluid. This model has advantages over the simpler power-law model, since most fluids exhibit a low shear-rate Newtonian plateau and a transition region into the power law regime. In the current formulation it is assumed that μ_∞ is zero. Therefore the constitutive equation becomes

$$\tau = \mu_0\left[1 + (\lambda\dot{\gamma})^2\right]^{(n-1)/2}\dot{\gamma} \tag{2}$$

For boundary layer flows the dominant term in the shear rate is $\dot{\gamma}$. Therefore the boundary layer equations over the plate are

$$\text{continuity equation:}\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$\text{momentum equation:}\ u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_0}{\rho}\left[1 + \left(\lambda\frac{\partial u}{\partial y}\right)^2\right]^{(n-1)/2}\frac{\partial^2 u}{\partial y^2} + \frac{\mu_0}{\rho}(n-1)\lambda^2\left(\frac{\partial u}{\partial y}\right)^2\left[1 + \left(\lambda\frac{\partial u}{\partial y}\right)^2\right]^{(n-3)/2}\frac{\partial^2 u}{\partial y^2} \tag{4}$$

where ρ is the fluid density. The boundary conditions for the free stream case (Blasius problem) are:

$$\text{At } y = 0: \quad u = 0, \quad v = 0 \tag{5}$$

$$\text{As } y \to \infty, \quad u = u_\infty \tag{6}$$

* Tel.: +30 25410 79618; fax: +30 25410 79616.
E-mail address: apantokr@civil.duth.gr

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Table 1
Physical properties of three silicone oils which behave as Carreau fluids [6].

Density, ρ (kg/m^3)	Dynamic viscosity, μ_0 (Pa s)	Relaxation time λ ($\times 10^{-3}$ s)	Power-law index, n
975	9.75	1.15	0.61
976	58.6	2.91	0.46
977	97.7	4.66	0.43

For the moving surface case (Sakiadis problem) we have

$$\text{At } y = 0: \quad u = u_w, \quad v = 0 \tag{7}$$

$$\text{As } y \to \infty, \quad u = 0 \tag{8}$$

where u_w and u_∞ are the velocities of the moving surface and the free stream respectively.

Introducing the following dimensionless quantities and notations

$$X = \frac{\rho u_{\text{ref}} x}{\mu_0}, \quad Y = \frac{\rho u_{\text{ref}} y}{\mu_0}, \quad U = \frac{u}{u_{\text{ref}}}, \quad V = \frac{v}{u_{\text{ref}}}, \quad De = \frac{\rho \lambda u_{\text{ref}}^2}{\mu_0} \tag{9}$$

the balance equations and the boundary conditions for the Blasius flow go over in the dimensionless forms

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{10}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \left[1 + \left(De\frac{\partial U}{\partial Y}\right)^2\right]^{(n-1)/2} \frac{\partial^2 U}{\partial Y^2} + (n-1)\left[1 + \left(De\frac{\partial U}{\partial Y}\right)^2\right]^{(n-3)/2} \left(De\frac{\partial U}{\partial Y}\right)^2 \frac{\partial^2 U}{\partial Y^2} \tag{11}$$

$$U(X, 0) = V(X, 0) = 0, \quad U(X, \infty) = 1 \tag{12}$$

while for Sakiadis flow the equations remain the same with boundary conditions

$$U(X, 0) = 1, \quad V(X, 0) = 0, \quad U(X, \infty) = 0 \tag{13}$$

taking into account that for Blasius flow $u_{\text{ref}} = u_\infty$ and for Sakiadis flow $u_{\text{ref}} = u_w$ respectively. In Eq. (9) X and Y are the longitudinal and transverse Reynolds number and De is the Deborah number. The wall shear stress (skin friction) is given by the equation

$$S = \left.\frac{\partial U}{\partial Y}\right|_{Y=0} = \frac{\mu_0}{\rho u_{\text{ref}}^2} \left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{14}$$

The Eqs. (10) and (11) represent a two-dimensional parabolic problem. Such a flow has a predominant velocity in the streamwise coordinate which is the direction along the plate. In this type of flow convection always dominates the diffusion in the streamwise direction. Furthermore, no reverse flow is acceptable in the predominant direction. The solution of this problem in the present work is obtained using a finite volume algorithm as described by Patankar [18]. In order to obtain complete form of velocity profiles at the same cross-section, a nonuniform lateral grid has been used. ΔY is small value near the surface (dense grid points near the surface) and increases with Y. A total of 500 lateral grid cells were used. It is known that the boundary layer thickness changes along X. For that reason, the calculation domain must always be at least equal to or wider than the boundary layer thickness. In each case, the goal was to have a calculation domain wider than the real boundary layer thickness. This has been done by trial and error. If the calculation domain was thin, the velocity and temperature profiles were truncated. In this case a wider calculation domain was used in order to capture the entire velocity and temperature profiles. The parabolic (space marching) solution procedure is described analytically in the textbook of Patankar [18]. That solution procedure is implicit and unconditionally stable [19], has been used extensively in the literature and has been included in fluid mechanics and heat transfer textbooks [19–21]. The method is used successfully also by others researchers (see for example, [22])

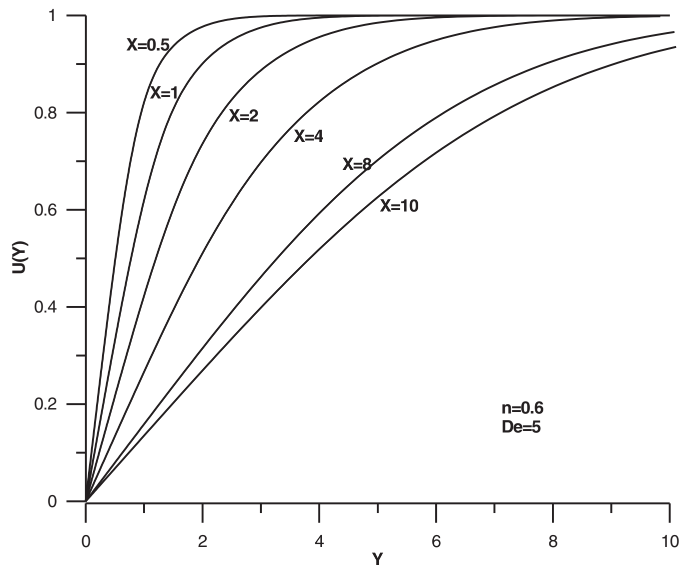

Fig. 1. Mainstream velocity profiles at different distances from the plate leading edge for $De = 5$ and $n = 0.6$ (Blasius flow).

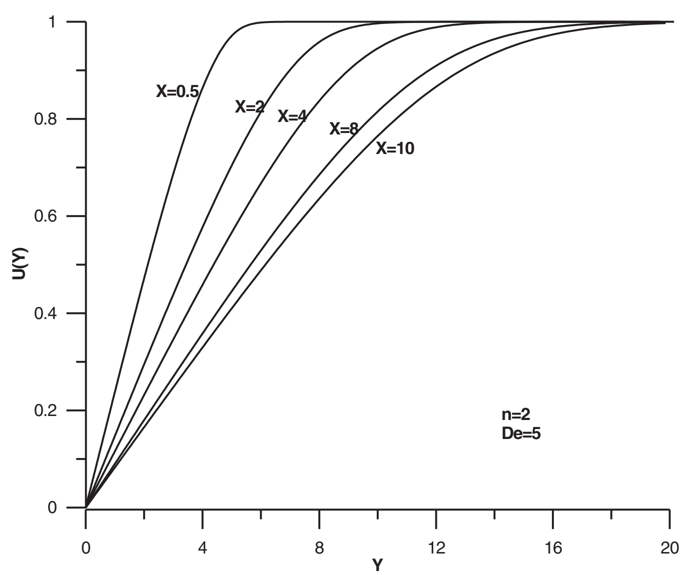

Fig. 2. Mainstream velocity profiles at different distances from the plate leading edge for $De = 5$ and $n = 2$ (Blasius flow).

3. Results and discussion

3.1. Blasius flow

Fig. 1 shows the variation of mainstream velocity at different non-dimensional distances along the plate for a shear-thinning fluid with $n = 0.6$ and Deborah number 5. It is seen that as X increases the velocity decreases and the boundary layer thickness gets higher values. In addition the wall shear stress declines continuously. The same behavior appears in Fig. 2 which concerns a shear-thickening