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Stagnation-point flow of a hydromagnetic viscous fluid over stretching/shrinking sheet with generalized slip condition in the presence of homogeneous–heterogeneous reactions

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ABSTRACT

An analysis is performed to study the effect of homogeneous–heterogeneous reactions on an electrically conducting viscous fluid near the stagnation-point past a permeable stretching/shrinking sheet with uniform suction and generalized slip condition. In this analysis we used the developed model of a homogeneous–heterogeneous reaction in boundary layer flow with equal diffusivities for reactant and autocatalysis. The governing partial differential equations in terms of continuity, momentum and concentration are transformed into ordinary differential equations and then solved numerically using shooting method. The results show that for the shrinking sheet, dual solutions exist in the certain range of involved parameters, while for stretching sheet, the solutions are unique. Comparison of the present results with previously published work is given and found in good agreement.

1. Introduction

The study of the stagnation-point flow of a viscous fluid over a stretching/shrinking sheet has gained attention of many researchers due to its wide range of applications in many industrial manufacturing processes. These processes include glass blowing, aerodynamic extrusion of plastic sheet, continuous casting and spinning of fibers, the cooling and drying of papers and textiles, etc. Hiemenz [1] is the first who has studied the two-dimensional stagnation flow past a fixed surface and obtained exact solution of the Navier–Stokes equations using similarity transformations. Paullet and Weidman [2] found the numerical solution of stagnation-point flow toward a stretching sheet. Sharma and Singh [3] investigated the effects of variable thermal conductivity and heat sink/source on MHD flow near a stagnation-point on a linearly stretching sheet. The steady two-dimensional MHD stagnation-point flow toward a stretching sheet with variable surface temperature was studied by Ishak et al. [4]. Hayat et al. [5] discussed the series solution of MHD two-dimensional mixed convection stagnation-point flow past a stretching vertical surface with thermal radiation in a porous medium. The effect of induced magnetic field on steady two-dimensional stagnation-point flow of an incompressible electrically conducting fluid toward a stretching sheet was analyzed by Ali et al. [6]. Subhashini et al. [7] developed the double diffusion convection near the stagnation – point region over a stretching sheet vertical surface with constant wall temperature. Bhattacharyya et al. [8] studied the reactive solute distribution in a laminar boundary layer stagnation-point flow of an electrically conducting viscous fluid over a stretching sheet with suction or blowing. Effects of variable thermal conductivity and heat source/sink on steady two-dimensional radiative MHD boundary layer flow of a viscous fluid in the presence of variable free stream near a stagnation-point over a stretching sheet was investigated by Al-Sudais [9]. On the other hand, it should be stated that flow due to a shrinking sheet was first studied by Miklavčič and Wang [10]. Further, Wang [11] investigated the stagnation-point flow of viscous fluid over a shrinking sheet and obtained a numerical solution for both two-dimensional and axisymmetric flows. Important and new results on the flow induced by a shrinking sheet in a viscous fluid near stagnation-point were discussed by Bhattacharyya and Layek [12], Bhattacharyya and Vajravelu [13], Ashraf and Ahmad [14], Rohni et al. [15] and Saleh et al. [16]. It is worth mentioning here that there are two conditions for the flow of a shrinking sheet to exist, namely, whether a sufficient suction is added on the boundary (Miklavčič and Wang [10]) or a stagnation flow is considered (Wang [11]) to maintain the velocity of shrinking sheet in the boundary layer.

In no-slip flow, the flow velocity at solid–fluid interface is zero but in slip flow, the flow velocity is non-zero at the solid wall. Navier [17] and Maxwell [18] investigated the linear slip boundary condition defining that the amount of relative slip is proportional to local

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shear stress, while the constant of proportionality is called slip length. Wang [19] presented an exact similarity solution of the Navier–Stokes equations for a stretching flow with partial slip. He discussed the perturbation and numerical solutions of the problem considering slip length to be constant. Motivated from molecular dynamical simulations, Thompson and Troian [20] suggested the generalized slip boundary condition, in which slip length is not a constant but a function of the shear stress. Also, Mathews and Hill [21] investigated the Newtonian flow with non-linear Navier boundary condition. Wang [22] analyzed the viscous flow due to a stretching sheet with slip and suction. Sajid et al. [23] solved numerically both planar and axisymmetric flow equations with general slip boundary condition.

In the recent years many researchers studied the stagnation-point flow with slip effects at the wall due to its applications in industrial processes. Aman et al. [24] analyzed the steady mixed convection boundary layer flow of a viscous fluid near the stagnation-point on a vertical surface with slip effects at the boundary. Bhattacharyya et al. [25] discussed the effects of partial slip on the steady boundary layer stagnation-point flow of a viscous fluid and heat transfer toward a shrinking sheet. Aman et al. [26] examined the stagnation-point flow of a viscous fluid in the presence of magnetic field with slip effects. The effects of variable fluid properties on heat and mass transfer in stagnation-point flow toward a vertical stretching sheet in the presence of surface slip was studied by Mostafa and Mahmoud [27]. Recently, Roşca et al. [28] presented results for the boundary layer flow and heat transfer of a viscous fluid near a stagnation-point flow on a non-linearly moving flat plate in a parallel free stream with partial slip velocity.

Chemical reactions can be classified as either homogeneous or heterogeneous process depending on whether they occur in bulk of the fluid (homogeneous) or occur on some catalytic surfaces (heterogeneous). Numerous chemically reacting systems contain both homogeneous and heterogeneous reactions such as in combustion, catalysis and biochemical systems. The correlation between homogeneous and heterogeneous reactions associated with formation and consumption of reactant species at different rates both within the fluid and on the catalytic surfaces is usually very complex. Chaudhary and Merkin [29–30] primarily constructed a simple model for homogeneous–heterogeneous reactions of stagnation-point flow in which homogeneous reaction is assumed to be given by an isothermal first order cubic kinetics and heterogeneous reaction, both for equal and unequal diffusivities. Later, Chaudhary and Merkin [31] extended their work to include the effect of loss of auto catalyst. Merkin [32] presented a model for the isothermal homogeneous–heterogeneous reactions considering the Blasius similarity solution. Khan and Pop [33] investigated the effects of suction/injection on steady-state boundary layer flow near the stagnation-point with homogeneous–heterogeneous reaction. Effect of homogeneous–heterogeneous reactions in stagnation-point flow on a stretching surface was studied by Bachok et al. [34]. Khan and Pop [35] investigated viscoelastic fluid flows past a stretching surface in the presence of a catalytic substance using an infinite difference method. Dual solution for stagnation-point flow of a nanofluid over a stretching/shrinking sheet in the presence of homogeneous–heterogeneous reactions was analyzed by Kameswaran et al. [36]. Further, Kameswaran et al. [37] investigated the effects of homogeneous–heterogeneous reactions over a stretching sheet in a porous medium saturated with a nanofluid. Shaw et al. [38] studied the effects of homogeneous–heterogeneous reactions on a boundary layer flow of a micropolar fluid over a permeable stretching/shrinking sheet in a porous medium.

The purpose of the current investigation is to study the MHD stagnation-point flow of an electrically conducting viscous fluid toward a permeable stretching/shrinking sheet with homogeneous–heterogeneous reactions and generalized slip effect at the boundary. Numerical solution of transformed similarity equations are obtained for both stretching and shrinking sheet in terms of velocity and concentration profiles for several values of the governing parameters.

2. Formulation of the problem

We consider the steady, two-dimensional and incompressible flow of an electrically conducting viscous fluid over a permeable stretching/shrinking sheet near a stagnation-point. The sheet is in the plane $y = 0$ and the stretched/shrunk sheet is in the $x-$direction with the velocity varying linearly along it *i.e.* $u_w(x) = mx$, where $m > 0$ is for the stretching sheet, $m = 0$ is for the static sheet and $m < 0$ is for the shrinking sheet, respectively. The free stream velocity is assumed as $u_e(x) = cx$, where $c > 0$ is the strength of the stagnation-point flow. A uniform external magnetic field of strength B_0 is applied normal to the stretching/shrinking surface. Under the assumption of small magnetic Reynolds number, the induced magnetic field is negligible and external electric field is zero. We also assume a simple model for the interaction between a homogeneous (or bulk) and heterogeneous (on sheet) reactions involving the two chemical species A and B in a boundary layer flow as stated by Chaudhary and Merkin [31] and Merkin [32]:

$$A + 2B \rightarrow 3B, \quad \text{rate} = k_c a b^2, \tag{1}$$

$$A \rightarrow B, \quad \text{rate} = k_s a. \tag{2}$$

where a and b are the concentrations of the chemical species A and B, respectively whereas k_i $(i = c,\ s)$ are the rate constant. We also expect here that both reaction processes are isothermal and far away from the sheet at the ambient fluid, there is a uniform concentration 'a_0' of reactant A and there is no auto catalyst in reactant B. Under usual boundary layer approximations and all the above mentioned assumptions, the governing equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}(u - u_e), \tag{4}$$

$$u\frac{\partial a}{\partial x} + v\frac{\partial a}{\partial y} = D_A\frac{\partial^2 a}{\partial y^2} - k_c a b^2, \tag{5}$$

$$u\frac{\partial b}{\partial x} + v\frac{\partial b}{\partial y} = D_B\frac{\partial^2 b}{\partial y^2} + k_c a b^2, \tag{6}$$

where u and v denote the velocity components in x and y directions, ν is the kinematic viscosity, σ is the electrical conductivity and, D_A and D_B are the respective diffusion coefficients. Following Thompson and Troian [20], we assume generalized slip boundary condition by the relation

$$u_t = \alpha^*(1 - \beta^*\tau_w)^{-\frac{1}{2}}\tau_w, \tag{7}$$

where u_t is the tangential velocity, α^* is the Navier's constant slip length, β^* is the reciprocal of some critical shear rate and τ_w is the shear rate at the wall. The boundary conditions for the problem under consideration are

$$\begin{gathered} u(0) = u_w(x) = mx + \alpha^*\left(1 - \beta^*(x)\frac{\partial u}{\partial y}\right)^{-\frac{1}{2}}\frac{\partial u}{\partial y}, \\ v(0) = -v_w, u(\infty) = u_e(x) = cx, \\ D_A\left.\frac{\partial a}{\partial y}\right|_{y=0} = k_s a(0), D_B\left.\frac{\partial b}{\partial y}\right|_{y=0} = -k_s a(0), a(\infty) = a_0, b(\infty) = 0. \end{gathered} \tag{8}$$

in which 'm' and 'c' are constants having dimension $(\text{time})^{-1}$, $v_w(> 0)$ is the constant mass transfer (suction) velocity and a_0 is a constant.