Journal of the Taiwan Institute of Chemical Engineers 54 (2015) 11–19

Contents lists available at ScienceDirect

Journal of the Taiwan Institute of Chemical Engineers

journal homepage: www.elsevier.com/locate/jtice

Approximate analytical modeling of heat and mass transfer in hydromagnetic flow over a non-isothermal stretched surface with heat generation/absorption and transpiration

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ARTICLE INFO

Article history:
Received 7 October 2014
Revised 13 March 2015
Accepted 21 March 2015
Available online 17 April 2015

Keywords:
Heat transfer
Mass transfer
Stretched surface
Skin friction
Nusselt number
HAM

ABSTRACT

This study reports the approximate analytical modeling of heat and mass transfer in hydromagnetic flow over a non-isothermal stretched surface, incorporating the effects of heat generation/absorption and transpiration. Homotopy analysis method (HAM) is used to obtain the analytical solutions of the non-linear governing equations. The effects of several parameters (suction/injection parameter f_w, magnetic parameter M, heat source/sink coefficient ε, Prandtl number Pr, Schmidt numberSc, and velocity and temperature power-law parameters $m,\ n$) on the dimensionless velocity, temperature, concentration, skin friction, and Nusselt and Sherwood numbers are investigated. The results of the local skin friction coefficient and reduced heat transfer rate are compared with the published data for a special case and found to be in good agreement. It is found that the reduced Nusselt and Sherwood numbers are increasing functions of f_w.

1. Introduction

The hydromagnetic flow and heat transfer due to a stretching surface have several engineering applications. For example, in the extrusion of a polymer sheet from a dye or in the drawing of plastic films, reduction of both thickness and width takes place. During the manufacturing process, the quality of the final product depends on the rate of heat transfer at the stretching surface [1]. Pavlov [2] examined the effect of magnetic field on flow characteristic over a stretching sheet and obtained an exact similarity solution of this problem. Chakrabarti and Gupta [3] reported the similarity solution for hydromagnetic flow over a stretching surface, whereas Chiam [4] extended the aforementioned research work of Chakrabarti and Gupta [3] to power-law velocity of the stretched surface using shooting method. Gupta and Gupta [5] and Ali [6] examined the effects of transpiration on flow field and heat transfer. With fourth-order Runge–Kutta–Merson method, the thermal analysis for the sheet due to power-law velocity was examined by Ali [6]. Heat generation effect on hydromagnetic flow and heat transfer was studied by Chamkha [7] with finite difference method. For various physical situations, the relative study of heat and/or mass transfer can be found in [8–16].

An extensive effort has been made to gain information regarding flow problems in various situations. Such situations include consideration of heat and mass transfer in magnetic field. A vast body of literature is now available on this topic. Ahmed and Sarmah [17] investigated MHD boundary layer flow and mass transfer over a plate. Afify [18] reported similarity solution in MHD effects of thermal diffusion on free convective heat and mass transfer over a stretching surface. Chamkha and Issa [19] investigated chemical reaction, heat generation or absorption effects on MHD boundary layer flow over a permeable stretching surface. Robert et al. [20] have discussed convective heat transfer in a conducting fluid over a permeable stretching surface with suction and internal heat generation/absorption. Ibrahim and Reddy [21] examined the radiation and mass transfer effects for MHD free convection flow along a stretching surface using shooting method.

So far, as we noticed, no studies have been carried out to obtain an analytical solution of hydromagnetic flow for heat and mass transfer over a non-isothermal stretched surface. In this study, we seek to obtain an approximate analytical solution of the non-linear ordinary differential equations which is derived from the similarity transformations. For validation of our analytical solution, results of skin friction factor and heat transfer are compared with those obtained by [4,7,8,14–16]. The results are found in good agreement and we believe that HAM gives accurate results.

The homotopy analysis method is an approximate analytical method introduced by Liao [22] to solve various non-linear

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http://dx.doi.org/10.1016/j.jtice.2015.03.022
1876-1070/

transport phenomena. After Liao [22], HAM has been successfully used to solve nonlinear problems in fluid mechanics and heat transfer. For instance, Dinarvand et al. [23] utilized HAM to study unsteady MHD flow close to a stagnation point of a rotating and translating sphere. Turkyilmazoglu [24] studied heat and mass transfer on MHD flow over a stretching sheet in the presence of hydrodynamic/thermal slip using HAM. Rashidi et al. [25] examined analytically, via HAM, laminar-free convective flow of a two-dimensional electrically conducting viscoelastic fluid over a moving stretching surface through porous medium. Masood et al. [26] used HAM to investigate MHD mixed convection Falkner–Skan flow with convective boundary conditions. Mabood and Khan [27] studied flow and heat transfer in Darcian porous medium with HAM. Ziabakhsh et al. [28] investigated stagnation point flow in porous media. Nadeem and Akbar [29] examined MHD heat and mass transfer on the peristaltic flow of a Johnson Segalman fluid in a vertical asymmetric channel using HAM while Mabood and Khan [30] have provided homotopy solution for heat transfer on MHD stagnation point flow in porous medium. These studies indicated the strength, effectiveness and flexibility of HAM to provide highly accurate analytical solutions for nonlinear problems.

In the present investigation, we studied heat and mass transfer on MHD flow over a non-isothermal stretched surface in the presence of suction/injection and heat generation/absorption. The considered problem is an extension of the paper of Chamkha [7], who studied only flow and heat transfer for hydromagnetic flow. The basic equations governing the flow are in the form of partial differential equations and have been reduced to a set of non-linear ordinary differential equations by using suitable similarity transformations. These equations are solved by homotopy analysis method. The expressions for dimensionless velocity, temperature and concentration are obtained. The effects of suction/injection (f_w), magnetic parameter (M), heat generation\absorption parameter (ε), Prandtl number (Pr), velocity exponent parameter (m), temperature exponent parameter (n) and Schmidt number (Sc) on the skin friction, dimensionless heat and mass transfer rates are studied.

2. Governing equations

Consider a two-dimensional steady, laminar, hydromagnetic flow of a quiescent electrically conducting fluid driven by a non-isothermal permeable stretched surface. The power-law velocity and temperature distributions are assumed in the analysis. A non-uniform transverse magnetic field is imposed normal to the flow direction. The magnetic field induced by the motion of the electrically conducting fluid and the pressure gradient are neglected. The governing equations for the continuity, momentum and energy can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial \mathrm{v}}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + \mathrm{v}\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \sigma\frac{B(x)^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{Q(x)}{\rho c_p}(T - T_\infty), \tag{3}$$

$$u\frac{\partial C}{\partial x} + \mathrm{v}\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

where x and y are the coordinates along and normal to the surface, u and v are the components of the velocity in the x- and y-directions respectively, ρ is fluid density, ν is kinematic viscosity, D is mass diffusivity, σ is the electrical conductivity, $B(x)$ is the variable magnetic field, α is the thermal diffusivity, c_p is specific heat at a constant pressure, $Q(x)$ is the heat generation/absorption coefficient, T is the temperature, and T_∞ is the ambient temperature of the fluid. The boundary conditions for the above model are [7]:

$$\begin{cases} y = 0: & u(x) = u_w(x) = U_0 x^m, \quad \mathrm{v} = \mathrm{v}_w(x), \\ & T = T_w(x) = ax^n + T_\infty, \quad C = C_w, \\ y \to \infty: & u(x) = 0, \quad T = T_\infty, \quad C = C_\infty, \end{cases} \tag{5}$$

where U_0, a, m and n are constants and $u_w(x)$, $\mathrm{v}_w(x)$, and $T_w(x)$ are the surface velocities along and normal to the surface and temperature at the wall, respectively.

In the usual manner, the stream function ψ is defined as $u = \partial\psi/\partial y$ and $\mathrm{v} = -\partial\psi/\partial x$ so that Eq. (1) is satisfied. Introduce the similarity transformations:

$$\eta = \sqrt{\frac{(m+1)x^m U_0}{2\nu x}}\,y, \quad \psi = \sqrt{\frac{2\nu U_0 x^{m+1}}{m+1}}\,f(\eta), \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$
$$\phi = \frac{C - C_\infty}{C_w - C_\infty}, \tag{6}$$

where η is the similarity variable. A similarity solution arises if the magnetic induction $B(x)$ and the heat generation/absorption $Q(x)$ take on the forms:

$$B(x) = B_0 x^{(m-1)/2}, \quad Q(x) = Q_0 x^{(m-1)}, \tag{7}$$

where B_0 and Q_0 are constants.

Under the transformations (6), the differential equations (2)–(4) reduce to

$$f''' + ff'' - Af'^2 - Mf' = 0, \tag{8}$$

$$\theta'' + Pr\left(f\theta' - \frac{A}{m}(nf' - \varepsilon)\theta\right) = 0, \tag{9}$$

$$\phi'' + Scf\phi' = 0, \tag{10}$$

with boundary conditions

$$f(0) = f_w\sqrt{\frac{2}{m+1}}, \quad f'(0) = 1, \quad f'(\infty) = 0, \quad \theta(0) = 1,$$
$$\theta(\infty) = 0, \quad \phi(0) = 1, \quad \phi(\infty) = 0. \tag{11}$$

where $Pr = \frac{\nu}{\alpha}$ is the Prandtl number, $Sc = \frac{\nu}{D}$ is the Schmidt number, $A = \frac{2m}{m+1}$ is a constant, $M = \frac{2\sigma B_0^2}{\rho(m+1)U_0}$ is the magnetic parameter, $\varepsilon = \frac{Q_0}{\rho c_p U_0}$ is the dimensionless heat generation/absorption coefficient, $f_w = -\mathrm{v}_w\sqrt{x^{1-m}/U_0\nu}$ is suction/injection parameter with, $f_w > 0$ representing suction, $f_w < 0$ representing injection and $f_w = 0$ corresponding to an impermeable surface. The prime denotes differentiation with respect to η.

The physical quantities of interest are the local skin friction C_f, the local Nusselt number Nu_x, and the local Sherwood number Sh_x. Physically, C_f represents the dimensionless wall shear stress, Nu_x and Sh_x define the dimensionless heat and mass transfer rates respectively.

$$C_f = \frac{\mu}{\rho u_w^2}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad Nu_x = \frac{-x}{T_w - T_\infty}\left(\frac{\partial T}{\partial y}\right)_{y=0},$$
$$Sh_x = \frac{-x}{C_w - C_\infty}\left(\frac{\partial C}{\partial y}\right)_{y=0}, \tag{12}$$

Using Eq. (6) into Eq. (11) to obtain the final dimensionless form

$$\sqrt{Re_x}C_f = \sqrt{\frac{m+1}{2}}f''(0), \quad \frac{Nu_x}{\sqrt{Re_x}} = -\sqrt{\frac{m+1}{2}}\theta'(0),$$
$$\frac{Sh_x}{\sqrt{Re_x}} = -\sqrt{\frac{m+1}{2}}\phi'(0), \tag{13}$$

where $Re_x = \frac{u_w x}{\nu}$ is local Reynolds number. From Eq. (13) we see that the local skin friction C_f, the local Nusselt number Nu_x, and the local Sherwood number Sh_x are proportional to the numerical values of $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ respectively.