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Entropy analysis of magnetohydrodynamic flow and heat transfer over a convectively heated radially stretching surface

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ABSTRACT

Steady boundary layer flow and heat transfer of an electrically conducting viscous fluid over a radially stretching surface is considered where the surface is convectively heated. Emphasis has been laid to study the entropy effects during the flow and heat transfer process. The non-linear partial differential equations governing the momentum and energy equations are converted into a set of non-linear ordinary differential equations by using suitable similarity transformations. The transformed system is then solved by using analytical and numerical techniques and the obtained results are compared with each other. The effects of various thermophysical parameters on velocity, temperature, local skin friction, Nusselt number, local entropy generation number and Bejan number are discussed in detail with the help of graphs and tables.

1. Introduction

Viscous fluid flow due to stretching surface is a subject of great interest for scientists and engineers due to its applications in many industrial and manufacturing processes such as extrusion of sheet materials, rolling of plastic films, coding of metallic plates, *etc.* Since the pioneer work done by Crane [1] on boundary layer flow caused by stretching of elastic flat surface, a large number of researchers have investigated this problem from different point of view of which some are mentioned in [2–15]. However, less attention has been paid on boundary layer flow and heat transfer over a radially stretching surface. Ariel [16] investigated the axisymmetric flow of a viscoelastic fluid over a radially stretching surface and obtained perturbation and approximate solutions of the problems. The axisymmetric, three dimensional stagnation point flow toward a sheet stretching in the radial direction in the presence of transverse magnetic field and heat generation effects was studied by Attia [17]. Wang [18] analyzed the natural convection flow on a vertical radially stretching surface and showed the asymptotic properties and uniqueness of the solution. Ganji et al. [19] made use of analytical technique known as Adopted Variational Itertative Method (AVIM) to examine the axisymmetric flow over a radially stretching surface. Fang [20] extended the axisymmetric stretching problem to a disk stretching and rotating at the same time. Shahzad et al. [21] gave the exact solution of axisymmetric flow over a nonlinear radially stretching sheet in terms of Gamma function. Recently, Shateyi and Makinde [22] discussed the hydromagnetic stagnation point flow toward a radially stretching disk which is convectively heated. They used Chebyshev spectral collocation method to obtain the solution of the problem.

Entropy analysis of flow and heat transfer systems is important as it identifies the factors which are responsible for the loss of useful energy. This energy loss can affect the efficiency of the thermally designed system. By reducing the factors that create entropy, the performance of the system can be enhanced. This idea was given by Bejan [23] which was later on adopted by other researchers and scientists. A lot of literature is available relating to the study of entropy generation effects in different geometrical configurations and situations [24–39].

The present article aims to investigate the entropy effects in magnetohydrodynamic flow and heat transfer over a convectively heated radially stretching disk. The effects of viscous dissipation and Joule dissipation are also taken into consideration. Analytical and numerical solutions of the problem are obtained and are discussed with the help of graphs and tables.

2. Mathematical formulation of the problem

Assume a steady two-dimensional boundary layer flow of an electrically conducting incompressible viscous fluid due to

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stretching of a convectively heated disk in the radial direction. The disk is stretching with velocity proportional to the distance from the origin, *i.e.*, $u = cr$, where c is a constant. The sheet is located in the plane $z = 0$ and the fluid is confined to the region $z > 0$. A uniform transverse magnetic field of strength B_0 is applied parallel to z-axis as shown in Fig. 1. It is considered that the magnetic Reynolds number is small so that the induced magnetic field is neglected. Also, there is no existence of electric field. The bottom surface of the disk is convectively heated by a hot fluid of temperature T_f which provides a heat transfer coefficient h. Then the governing equations of fluid flow and heat transfer are:

$$\frac{\partial}{\partial r}(ru) + \frac{\partial}{\partial z}(rw) = 0, \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B_0^2}{\rho}u, \tag{2}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{\sigma B_0^2}{\rho c_p}u^2, \tag{3}$$

with the boundary conditions

$$\begin{aligned} &u = u_w(r) = cr, \quad w = 0, \quad -k\frac{\partial T}{\partial z} = h(T_f - T) \quad \text{at}\, z = 0, \\ &u \to 0, \quad T = T_\infty \quad \text{at}\, z \to \infty. \end{aligned} \tag{4}$$

where u and v are the velocity components in the radial and axial directions respectively, ν is the kinematic viscosity of the fluid, σ is the electrical conductivity of the fluid, B_0 is the applied magnetic field, ρ is the density of the fluid, c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, T is the temperature of the fluid, T_f is the hot fluid temperature, T_∞ is the temperature far away from the surface and c is the dimensional constant.

In order to non-dimensionalize Eqs. (1)–(3), following similarity transformations are introduced:

$$u = crf'(\eta), \quad v = -\sqrt{c\nu}f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \eta = \sqrt{\frac{c}{\nu}}z. \tag{5}$$

By substituting (5) into Eqs. (1)–(3), Eq. (1) is automatically satisfied and Eqs. (2) and (3) take the form

$$f''' + 2ff'' - f'^2 - Mf' = 0, \tag{6}$$

$$\theta'' + 2\,Pr\,f\theta' + Pr\,Ec\,f''^2 + M\,Pr\,Ec\,f'^2 = 0, \tag{7}$$

where $M = (\sigma B_0^2/\rho c)$ is the magnetic parameter, $Pr = (\mu c_p/k)$ is the Prandtl number and $Ec = (u_w^2/(c_p(T_f - T_\infty)))$ is the Eckert

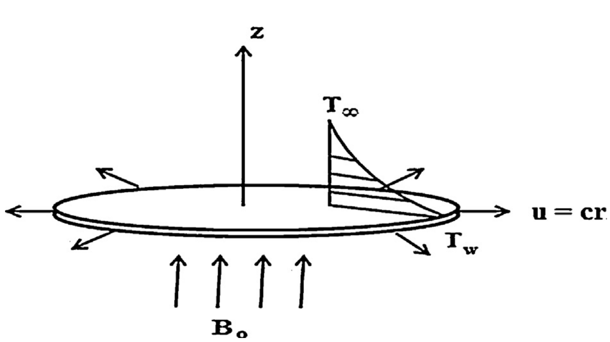

Fig. 1. Schematic diagram of the problem.

number. Making use of (5), the boundary conditions (4) become

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = -Bi(1 - \theta(0)), \\ &f'(\infty) = 0, \quad \theta(\infty) = 0. \end{aligned}$$

Here $Bi = (h/k)\sqrt{\nu/a}$ is the Biot number. The expressions of local skin friction coefficient C_f and local Nusselt number Nu are given by expressed as

$$C_f = \frac{\tau_w}{(1/2)\rho u_w^2}, \quad Nu = \frac{rq_w}{k(T_f - T_\infty)}, \tag{9}$$

where τ_w is the shear wall stress and q_w is the heat flux from the surface and are given by

$$\tau_w = \mu\left(\frac{\partial u}{\partial z}\right)_{z=0}, \quad q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=0}. \tag{10}$$

Substituting (10) in (9) and after using (5), the non-dimensional forms of local skin friction coefficient and local Nusselt number are

$$\frac{1}{2}C_f Re_r^{1/2} = -f''(0), \quad Re_r^{1/2}Nu = -\theta'(0), \tag{11}$$

3. Entropy generation

Using boundary layer approximation, the local volumetric rate of entropy generation S_G for a viscous fluid in the presence of magnetic field is defined by [23]:

$$S_G = \underbrace{\frac{k}{T_f^2}\left(\frac{\partial T}{\partial z}\right)^2}_{\substack{\text{entropy effects} \\ \text{due to} \\ \text{heat transfer}}} + \underbrace{\frac{\mu}{T_f}\left(\frac{\partial u}{\partial z}\right)^2}_{\substack{\text{entropy effects} \\ \text{due to} \\ \text{fluid friction}}} + \underbrace{\frac{\sigma B_0{}^2}{T_f}u^2}_{\substack{\text{entropy effects} \\ \text{due to} \\ \text{magnetic field}}}. \tag{12}$$

The dimensionless form of entropy generation is expressed as

$$Ns = \frac{S_G}{S_o} = Re_L\theta'^2 + Re_L\frac{Br}{\Omega}f''^2 + Re_L\frac{Br}{\Omega}Mf'^2, \tag{13}$$

where $S_o = ((k(T_f - T_\infty)^2)/T_f{}^2L^2)$ is the characteristic entropy generation rate, $\Omega = ((T_f - T_\infty)/T_f)$ is the dimensionless temperature difference, $Re_L = (u_L L/\nu)$ is the Reynolds number and $Br = Pr\,Ec$ is the Brinkman number. Thus, local entropy generation in Eq. (13) can be written as:

$$Ns = N_H + N_f + N_m = N_H + N_F, \tag{14}$$

where $N_F = N_f + N_m$. Here N_H is the local entropy generation due to heat transfer, N_f is the local entropy generation due to fluid friction and N_m is the local entropy generation due to magnetic field.

Another important irreversibility distribution parameter is known as Bejan number and is defined as follows:

$$Be = \frac{N_H}{Ns}. \tag{15}$$

From (15), it is quite evident that value of the Bejan number ranges from 0 to 1. When $Be > 0.5$, the heat transfer entropy effects are in dominance whereas for $Be < 0.5$, entropy effects due to viscous dissipation and magnetic field are dominating. The contribution of entropy due to heat transfer is equal to that of fluid friction and magnetic field, when $Be = 0.5$.

4. Solution of the problem

The set of equations (6) and (7) together with the boundary conditions (8) are coupled nonlinear equations which are solved by