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Finite element modeling of a double-diffusive mixed convection flow of a chemically-reacting magneto-micropolar fluid with convective boundary condition

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ABSTRACT

In this paper, the steady, two-dimensional, heat and mass transfer flow of a chemically reacting mixed convection magneto-micropolar fluid over a wedge with a convective surface boundary condition is investigated. Using similarity transformations found by Falkner and Skan, the governing transport equations are reduced to a system of non-linear ordinary differential equations which are solved by employing the extensively-validated, finite element method. The results are depicted graphically to illustrate the influence of the various pertinent parameters on velocity, micro-rotation, temperature and concentration functions. Additionally, skin friction coefficient, local Nusselt number and local Sherwood number are also computed and presented graphically for the flow regime. The numerical solutions are compared with earlier studies for special cases of the wedge angle parameter and found to be in excellent agreement, thereby validating the accuracy of the present numerical code. The study finds applications in chemical reaction engineering processes, magnetic materials processing, solar collector energy systems, etc.

1. Introduction

Flows with chemical reaction abound in many branches of engineering science and geophysics including food processing, cooling towers and chemical engineering processes [1]. In such flows, the chemical reaction may either occur uniformly throughout a given phase (homogeneous reaction) or in a restricted region (boundary) of the phase (heterogeneous reaction). Combined heat and mass transfer flows with chemical reactions have stimulated extensive research in science and technology in the past few decades due to numerous applications in drying processes, combustion processes, metallurgical flows, cooling towers, etc. In this regard, a number of studies involving many multi physical flow phenomena have been communicated by many authors [2–6] using the boundary layer theory. Significant analysis of electrically conducting convective flows with chemical reaction, have also received attention owing to diverse applications in nuclear fusion, chemical engineering, magnetic materials processing, astrophysical flows, plasma studies, geophysics and hybrid MHD power generators. In the past decade, many authors [7–10] have explored various aspects of heat and mass transfer in such types of flows. An excellent discussion of the many important works is available in the monograph by Gebhart et al. [11].

The above studies are all confined to the Navier–Stokes fluid model. However, in various chemical engineering applications, materials processing engineering, biomechanics, slurry technologies etc, the fluid used exhibit microstructural characteristics *i.e.* rotary motions and also gyration of fluid microelements. Eringen in his pioneering paper [12] formulated the micropolar fluid model to simulate such effects. The micropolar model takes into account the inertial characteristics of the substructure particles which are allowed to sustain rotation and couple stress. Such type of flows find applications in the purification of crude oil, polymer technologies, cooling tower dynamics, chemical reaction engineering, metallurgical drawing of filaments and solar energy systems. Eringen [13] later extended the theory to include thermal effects and developed the theory of thermo-micropolar fluids. Numerous studies investigating various aspects of micropolar fluid flows have therefore been subsequently reported [14–18]. An

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Nomenclature

Roman

b	constant
C	concentration of species in the boundary layer ($kmol/m^3$)
D	chemical molecular diffusivity (m^2/s)
k_1	rate of chemical reaction (mol/m/s)
f	dimensionless stream function
g	dimensionless micro-rotation
g_e	acceleration due to gravity (m/s^2)
h	dimensionless velocity
h_f	heat transfer coefficient ($W/m^2/K$)
i	dimensionless microinertia
j	microinertia per unit mass (kg/m^3)
k_f	thermal conductivity of the fluid (W/m/K)
N	micro-rotation component ($kg\ m^2/s$)
T	temperature of the fluid in the boundary layer (K)
$U(x)$	free stream velocity (m/s)

Greek

β	thermal expansion coefficient (K^{-1})
β^*	concentration expansion coefficient (K^{-1})
σ	electrical conductivity ($s^3\ A^2/kg/m^3$)
η	dimensionless coordinate
θ	dimensionless temperature
ϕ	Dimensionless concentration
α	Thermal diffusivity (m^2/s)
ρ	Fluid density (kg/m^3)
μ	Dynamic viscosity (kg/m/s)
ν	Kinematic viscosity (m^2/s)
κ	micro-rotation viscosity (m^2/s)
γ	spin gradient viscosity (m^2/s)
Ψ	stream function (m^2/s)
τ_w	shear stress on the wall (Pa)

Subscripts

w, f	condition at the surface and of the hot fluid, respectively
∞	condition far away from the surface

Superscript

$'$	differentiation with respect to η

excellent discussion of the applications of micropolar fluids has been communicated by Ariman et al. [19] and very recently in the monograph by Bég et al. [20].

Transport phenomena along a wedge, from technological point of view, is of special interest and has many engineering applications in chemical engineering, aerodynamics, heat exchanger, storage of nuclear waste etc. In these flows, the free stream velocity is proportional to a power of the length coordinate measured from the stagnation point. The laminar flow past such a body was first analyzed by Falkner and Skan [21] to illustrate the application of Prandtl's boundary layer theory. Using similarity transformation, they reduced the boundary layer equation to an ordinary differential equation, which is well-known as the Falkner–Skan equation. The solutions of Falkner–Skan equations are sometimes referred to as wedge-flow solutions. Later, Hartree [22] obtained the numerical solution of this equation. Thereafter, the problem has been studied extensively in various aspects of Newtonian fluids [23–28]. Micropolar transport past a wedge has also received some consideration. Nath [29] obtained similar solutions for the steady laminar incompressible boundary layer equations of micropolar fluid in the stagnation region of an infinite wedge. Kim [30] examined the steady laminar flow of micropolar fluids past a wedge with constant surface temperature. Ishak et al. [31] obtained the similarity solutions for the boundary layer flow over a moving wedge and past a flat plate in a micropolar fluid. Ishak et al. [32] further investigated the boundary layer flow of a micropolar fluid past a wedge with constant surface heat flux and in the presence of variable magnetic field.

In all of the above mentioned studies, the convective heat exchange at the surface was not considered. In many practical applications involving cooling or heating of the surface, the presence of convective heat exchange between the surface and the surrounding fluid cannot be neglected. Such flow problems are important in engineering and industrial processes like transpiration cooling process, material drying, heat exchangers, etc. Several articles with convective boundary condition have been reported [33–37]. Keeping the above facts in view, the present work investigates the double-diffusive mixed convection flow of a chemically reacting magneto-micropolar fluid over a wedge with a convective boundary surface condition. A finite element solution is presented to study the effect of various key physical parameters which control the flow regime. This study constitutes a new flow model and thus far has not been reported in the technical literature.

2. Mathematical formulation

Consider the steady, two-dimensional, incompressible, laminar, mixed convection boundary-layer flow of an electrically conducting magneto-micropolar fluid over a wedge heated by convection from a hot fluid at temperature T_f with a heat transfer coefficient h_f in the presence of first-order chemical reaction. The physical model and the coordinate system are shown in Fig. 1. The x-axis is measured along the surface of the wedge and y-axis normal to it. Let u and v be the velocity components along x and y directions, respectively. A variable magnetic field $B(x)$ of the form $B(x) = B_0 x^{(m-1)/2}$ is applied parallel to the y-axis. This form of $B(x)$ admits similarity solutions [32]. The magnetic Reynolds number of the flow is taken to be small enough so that the induced magnetic field is negligible. The concentration of the species, C_w and the temperature, T_w at the wedge surface are assumed to be greater than the ambient temperature and concentration, T_∞ and C_∞, respectively. The fluid properties of the flow model are assumed to be independent of temperature and chemical species concentration

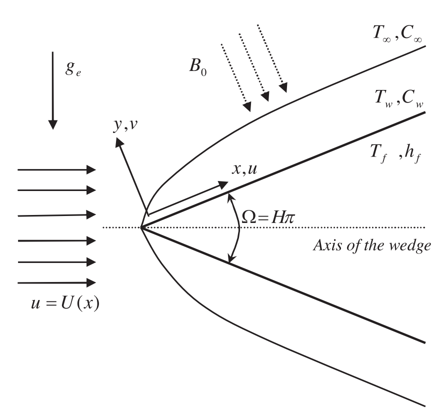

Fig. 1. Physical model and coordinate system.

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