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Radiation effects in three-dimensional flow over a bi-directional exponentially stretching sheet

M. Mustafa [a], A. Mushtaq [b,*], T. Hayat [c,d], A. Alsaedi [d]

[a] School of Natural Sciences (SNS), National University of Sciences and Technology (NUST), Islamabad 44000, Pakistan
[b] Research Centre for Modeling and Simulation (RCMS), National University of Sciences and Technology (NUST), Islamabad 44000, Pakistan
[c] Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan
[d] Department of Mathematics, Faculty of Science, King Abdulaziz University, PO Box 80257, Jeddah 21589, Saudi Arabia

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ABSTRACT

Analysis is performed to explore the characteristics of non-linear radiation heat transfer in the three-dimensional flow above an exponentially stretching sheet. The temperature is also exponentially distributed across the sheet. Local similarity solutions of the arising differential equations are computed through shooting method with fourth-fifth-order Runge–Kutta integration technique. The results corresponding to different values of temperature exponent parameter (A) reveal some interesting facts about the temperature distribution. The fluid's temperature is largely influenced with the variations in wall to ambient temperature ratio. The solutions are found in excellent agreement with the existing studies in the literature.

1. Introduction

The fundamental work of Blasius [1] on the laminar boundary layer flow over a flat plate has played a vital role in the advancements of fluid mechanics during the last century. Sakiadis [2] discussed the flow over a moving plate in an otherwise ambient fluid. Crane [3] extended the Sakiadis problem for a stretching sheet and analytically examined the flow and heat transfer characteristics by using similarity transformations. Ever since various aspects of these classical problems have been reported by researchers such as the influences of body forces due to electromagnetic or gravitational potentials, viscoelastic effects, heat and mass transfer effects, impact of porosity or permeability *etc.* (see [4–11]). Most of these problems were confined to the boundary layers above moving surfaces with constant or linearly stretching velocity. However, not all the industrial applications deal with the surfaces those have linear or constant stretching velocity. Keeping this in view, Magyari and Keller [12] used exponentially stretching continuous surface with exponential surface temperature distribution to examine the flow and heat transfer characteristics. Sajid and Hayat [13] studied the effect of thermal radiation on boundary layer flow induced due to exponentially stretching sheet. They obtained series expressions of velocity and temperature functions by homotopy analysis method (HAM). Khan and Sanjayanand [14] obtained an approximate analytic solution for viscoelastic flow above an exponentially stretching sheet. Nadeem and Lee [15] discussed the flow of nanofluid over an exponentially stretching wall by HAM. Ferdows et al. [16] investigated the mixed convection flow of electrically conducting nanofluid past an exponentially stretching immersed in a porous medium. Recently, Mustafa et al. [17] provided analytic solution for flow of nanofluid over an exponentially stretching sheet with convective boundary conditions. In another paper Mustafa et al. [18] derived both numerical and series solutions for stagnation-point flow of nanofluid past an exponentially stretching sheet. In recent work of Liu et al. [19] three-dimensional boundary-layer flow and heat transfer characteristics driven by a bi-directional exponentially stretching horizontal surface is considered. Numerical and analytic solutions for two-dimensional flow of Powell–Eyring fluid were developed by Mushtaq et al. [20]. Beg et al. [21] discussed the explicit numerical solution of an unsteady nanofluid flow due to exponential stretching in porous medium under the influence of buoyancy and magnetic forces.

The study of thermal radiation is important in solar power technology, nuclear plants, propulsion devices for aircraft, combustion chambers and chemical processes at high operating temperature. Radiation effects in viscoelastic boundary layer flow

* Corresponding author. Tel.: +92 51 90855736.
E-mail addresses: ammar.mushtaq@yahoo.com, ammar.mushtaq@rcms.nust.edu.pk (A. Mushtaq).

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Nomenclature

u, v, w	velocity components along the x-, y- and z-directions
f, g	dimensionless x- and y-components of velocity
T	local fluid temperature
Pr	Prandtl number
T_w	sheet temperature
Re	local Reynolds number
T_∞	ambient fluid temperature
R_d	radiation parameter
U_0, V_0	reference velocities

Greek symbols

A	temperature exponent parameter
η	similarity variable
C_p	heat capacity of the fluid
θ	dimensionless temperature
q_r	radiative heat flux
μ	dynamic viscosity
k^*	mean absorption coefficient
ν	kinematic viscosity
q_w	sheet heat flux
ρ	density
k	thermal conductivity
α	thermal diffusivity
θ_w	temperature ratio parameter
σ^*	Stefan–Boltzman constant
L	reference length
λ	velocity ratio
C_{fx}, C_{fy}	Skin friction coefficients along the $x-$ and $y-$ directions
τ_{wx}, τ_{wy}	shear stresses along the $x-$ and $y-$ directions
Nu_x	local Nusselt number

using linearized Rosseland radiative heat flux have been discussed by Raptis and Perdikis [22]. Seddeek [23] numerically discussed the free convection flow past a vertical plate under the influence of radiation. The classical Sakiadis flow problem subject to radiation heat transfer from an isothermal plate was considered by Cortell [24]. Hayat et al. [25] obtained homotopy based series solutions for flow of viscoelastic (second grade) fluid and radiation heat transfer with viscous dissipation. EL-Hakiem [26] described the free convection flow of an electrically conducting fluid with thermal radiation and mass transfer effects. Boundary layer flow of nanofluid past a porous plate with viscous dissipation has been studied by Motsumi and Makinde [27]. Effects of radiation on the flow of nanofluid over a stretching sheet with power-law stretching velocity have been examined by Hady et al. [28]. Series solutions for flow of micropolar fluid under the impact of radiation have been computed by Shadloo et al. [29]. Behavior of incident radiation on the flow of nanofluid with a possible application to solar energy has been described by Kandasamy et al. [30]. Unsteady natural convection flow of nanofluid with combined heat and mass transfer in the presence of thermal radiation is recently discussed by Turkyilmazoglu and Pop [31]. Numerical solution for free convective flow of nanofluid due to porous stretching sheet is discussed by Ferdows et al. [32]. Khan et al. [33] explored the magnetic field effects on the flow of nanofluid over a radiative surface. In another paper, Khan et al. [34] discussed the effects of heat generation, radiation and chemical reaction on the flow of nanofluid past a wedge.

This article describes the steady three-dimensional boundary layer flow of a viscous fluid over a horizontal sheet which is exponentially stretched in two-lateral directions. Heat transfer analysis subject to non-linear radiative heat transfer is performed. It should be noted here that extensive literature pertaining to linear thermal radiation in steady/unsteady boundary layer flows is available. On the other hand the analysis of nonlinear radiation heat transfer has recently been introduced by some researchers (see Pantokratoras and Fang [35], Mushtaq et al. [36], Cortell [37] and Mushtaq et al. [38]. We assumed that temperature on the surface of stretching sheet is distributed exponentially. The developed nonlinear differential system has been solved numerically by shooting method using fourth-fifth-order-Runge–Kutta integration scheme. A comparative study of present results and the previously published ones is presented. The numerical results for wall heat flux have been discussed in detail.

2. Problem formulation

We consider the viscous incompressible flow over a plane elastic sheet stretched exponentially in two lateral directions. The sheet is aligned with the xy-plane ($z = 0$). Let $U_w(x,y) = U_0 e^{((x+y)/L)}$ and $V_w(x,y) = V_0 e^{((x+y)/L)}$ be the velocities of the stretching sheet along the x- and y-directions respectively. Heat transfer analysis is carried out in the presence of thermal radiation. The sheet is maintained at temperature $T_w = T_\infty + T_0 e^{A((x+y)/2L)}$ where T_∞ denotes the ambient fluid temperature and A is the temperature exponent parameter (see Fig. 1). Following Magyari and Keller [12] and Liu et al. [19], we will consider both positive and negative values of A in our analysis. Under the usual boundary layer assumptions, the equations governing the three-dimensional incompressible flow and heat transfer are (see Liu et al. [19])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu\frac{\partial^2 u}{\partial z^2}, \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu\frac{\partial^2 v}{\partial z^2}, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = a\frac{\partial^2 T}{\partial z^2} - \frac{1}{\rho C_p}\left(\frac{\partial q_r}{\partial z}\right), \tag{4}$$

where ν is the kinematic viscosity, u, v and w are the velocity components along the x-, y- and z-directions, respectively, ρ is the

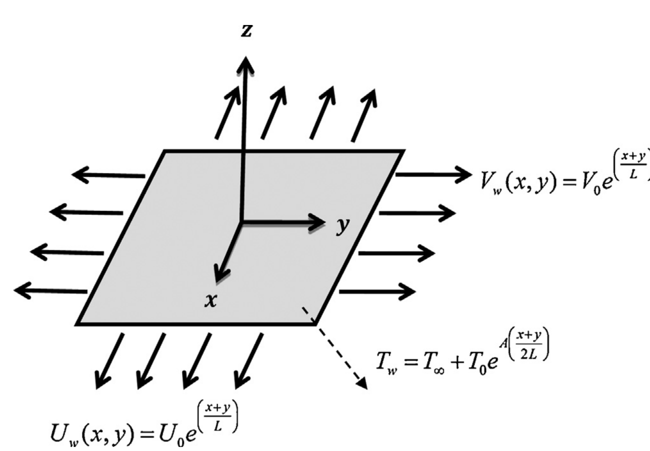

Fig. 1. Physical configuration and coordinate system.