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Marangoni instability of immiscible liquid–liquid stratified flow with a planar interface in the presence of interfacial mass transfer

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ABSTRACT

Marangoni instability of immiscible liquid–liquid stratified flow with interfacial mass transfer was investigated. In this approach, a linear approximation relation of surface tension with solute concentration was assumed, and the stability of the flow was studied by small disturbance theory. Numerical results indicated that Marangoni instability occurs when *Re* number is very small (less than 1) and it should be considered in liquid–liquid stratified flows with interfacial mass transfer.

1. Introduction

In recent years, liquid–liquid microextraction has been studied by many researchers for its good performance of rapid mass transfer, high extraction efficiency, small reagent consumption, convenient operation, etc. Because Marangoni instability is always induced by the interfacial tension gradient due to concentration disturbance, it has attracted lots of researchers since Pearson [1] and Sternling and Scriven [2].

Marangoni instability in liquid/liquid systems occurs with the presence of bulk concentration gradient normal to the interface. It is of interest since the interface instabilities promote the interfacial mass transfer significantly [3]. A lot of researchers focus on applying Marangoni instability to regulate the operation process. The transient drop rise velocity in the system of toluene/acetone/water for different initial solute concentrations and drop diameters were investigated by Wegener et al. [4]. Their results showed that the reacceleration time as an indicator of the end of Marangoni dominance can be expressed as a function of drop diameter and initial solute concentration. The deformable single droplet in the toluene/acetone/water system was experimental investigated by Wegener and Paschedag [5]. Their results revealed that the influence of deformation on the enhancement of mass transfer in Marangoni dominated systems.

Besides, the criterion of Marangoni instability was widely studied [6,7]. The classical linear instability analysis was carried by Sternling and Scriven [2]. Their approach is considered as the corner stone of theoretical research. The instability of various binary liquid–liquid interfaces in the presence of surfactant transferring was studied by Agble and Mendes-Tatsis [6]. The available criteria of instability for the surfactant transfer through a liquid interface were discussed by Kovalchuk and Vollhardt [8]. Their results indicated that the instability criteria are different for plane and spherical interfaces, the surface area and the depth of liquid layers can affect the instability, and the correlation between the rates of diffusion and adsorption is important to instability. The onset of Marangoni instability in partially miscible liquid–liquid systems in the presence of surface-active solutes was studied by Slavtchev et al. [9]. Their results indicated that the stability conditions depend mainly on physical parameters of the system, such as the solute diffusivity ratio and the kinematic viscosity ratio. The influence of low frequency vibration on Marangoni instability in a layer of binary mixture was studied by Fayzrakhmanova et al. [10]. Their results indicated that the vibration destabilizes the layer and the instability takes place even for zero Marangoni number. The above approaches only discuss Marangoni instability and rarely considering the important effects of mean flow.

Linear stability of two superposed fluids in plane Poiseuille flow was first studied theoretically by Yih [11], who derived a general expression for two-dimensional and long-wavelength disturbance waves. It was shown that the viscosity stratification can give rise to flow destabilization. The growth of disturbance depends on the geometric and physical parameters of flow system, such as the

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ω	wave speed
c	concentration
F	non-dimensional gravity parameter
C	mean concentration
C_{max}	the maximum value of mean concentration
D	solute diffusivity
H	half channel height
μ	dynamic viscosity
ν	kinematic viscosity
m	viscosity ratio
n	the solute concentration ratios of upper and lower liquid at the interface
p	the location of interface
r	density ratio
Re	Reynolds number
Re_c	critical Reynolds number
Ma	Marangoni number
Sc	Schmidt number
Pe	Peclet number
S	non-dimensional surface tension parameter
U	mean streamwise velocity
U_{max}	maximum value of mean velocity
V	mean velocity normal to the upper and below wall
u	flow streamwise velocity
v	flow velocity normal to the upper and below wall
N–S	Navier–Stokes equations
ΘH	the disturbance of interface position deviate from $y = pH$
x,y	Cartesian coordinates
Greek letters	
α	wave number
ρ	density
λ	diffusion ratios
σ	surface tension
Subscripts	
i	$i = 1$ upper layer or wall; $i = 2$ lower layer or wall
c	critical
s	disturbance

ratio of viscosity, density and layer thickness. The effect of viscosity ratio, thickness ratio, and surface tension on the stability of plane Poiseuille stratified flow was studied by Yiantsios and Higgins [12]. They predicted theoretically the critical Reynolds number of interfacial mode and shear mode. The two modes controlling the stability of stratified flow were verified by Hooper [13].

Deformable surface and Marangoni instability are always coexistence. How periodic wave trains are generated in liquid layers was studied by Linde et al. when a surface-active vapor is absorbed at a higher surface tension liquid [14]. Their results indicated that traveling periodic wave trains are excited beyond the critical Marangoni number. A stability due to a chemical absorption for an infinite liquid layer with a deformable free surface was investigated theoretically by Kozhoukharova and Slavchev [15,16]. Their results indicated that the surface deformability leads to the coexistence of oscillatory and stationary instability mode.

The stability of a plane interface with a linear concentration gradient and finite thickness of the both fluid layers was investigated by Reichenbach and Linde [17]. In their research, the effects of adsorption, surface viscosities and surface diffusion were not considered. Their results indicated that the critical Marangoni numbers were determined as a function of different parameters of both phases. A numerical study of two-dimensional Marangoni instability and shear instability of two-layer liquid flow was conducted by Mao et al. [18]. In their research, the linear relationship of interfacial tension versus the solute concentration was assumed and incorporated into a mathematical model accounting for liquid flow and mass transfer in both phases. Their results agree well with experiments. Two superposed liquid–liquid flow are widely used in the process of chemical engineering. However, the effects of fluid flow were seldom introduced when Marangoni instability was investigated in previous research. For this reason, the effects of flow were considered to investigate Marangoni instability of two superposed liquid–liquid plane Poiseuille flow in this paper. The results indicated that Marangoni instability occurs at very low Reynolds number ($Re < 1$) in the system of liquid–liquid stratified flow with interfacial mass transfer and the flow has large effects on the critical Marangoni number.

2. Theory

Marangoni effect is an interface convection phenomenon driven by interface tension gradient at the interface of two contact fluids, and the strength of it is determined by dimensionless Marangoni number *Ma*. To study the mechanism of Marangoni instability, the following assumptions were made for simplification:

(1) The fluids are Newtonian, incompressible, viscous and isothermal liquids;
(2) The physical properties of fluids are constants;
(3) An approximate linear relation of surface tension with solute concentration was assumed: $\Delta\sigma = \Delta C d\sigma/dC$, usually $d\sigma/dC < 0$;
(4) The two liquids are immiscible and the thickness of interface layer is infinitesimal;
(5) The two fluids are of equal densities but different viscosities, and no slip at the interface.

Although the surface tension is generally a non-linear function of solute concentration, the linear approximation is reasonable when the concentration difference is small for assumption (4).

Fig. 1 shows the schematic of flow configuration. The origin of the vertical coordinate was located at the center of the channel. All quantities were nondimensionalized by half channel height H, the maximum value of mean velocity U_{max} and concentration C_{max}, and the density ρ_2, viscosity μ_2, solute diffusivity D_2 of the below fluid.

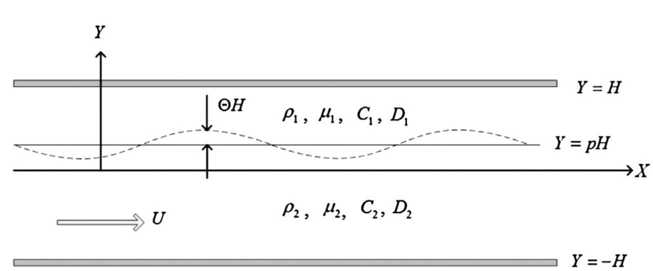

Fig. 1. The schematic of a two-layer flow. Where, the interface is planar and nondeformable at original steady state, and the dotted line represents the disturbance ΘH.