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Numerical investigation on mixed convective peristaltic flow of fourth grade fluid with Dufour and Soret effects

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ABSTRACT

Heat and mass transfer analysis in the mixed convective peristaltic flow of fourth-grade fluid under viscous dissipation, Dufour and Soret effects is carried out. Mathematical model is formulated by incorporating long wavelength and low Reynolds number assumptions. The resulting coupled nonlinear boundary value problem (BVP) has been solved numerically by Keller–box method. The computations are validated through the built in routine for solving nonlinear boundary value problems via shooting method through the software Mathematica. The results indicate an increase in the pumping rate and a decrease in the temperature and concentration functions with an increase in the elastic parameter (Deborah number) for fourth grade fluid. The temperature and concentration are increasing functions of the buoyancy forces due to temperature and concentration gradients.

1. Introduction

Peristalsis is the form of fluid transport due to the wave travelling along the walls of an inextensible tube/channel. Such fluid transport occurs in uterus, lower intestine, cervical canal, gastrointestinal tract, female fallopian tube, lymphatic vessels and small blood vessels. This phenomenon also occurs in the roller and finger pumps to pump the noxious fluid in nuclear industry. The pioneering work on the peristaltic mechanism was reported by Latham [1] and Jaffrin and Shapiro [2]. In the past, the peristaltic flows of differential type fluids have been given significant attention. For example, Haroun [3] analyzed the peristaltic motion of third order fluid in an asymmetric planar channel. This work in the presence of slip condition has been studied by Hayat *et al.* [4]. In another paper, nonlinear peristaltic flow of fourth grade fluid has been examined by Haroun [5]. Wang *et al.* [6] have discussed the influence of slip on peristaltic flow of third grade fluid in a circular tube. Nadeem *et al.* [7] investigated the characteristics of heat and mass transfer on the peristaltic flow of third order fluid in a diverging channel. Ali *et al.* [8] examined the magnetic field effects on the peristaltic motion of third grade fluid in an asymmetric planar channel. Peristaltic flow of third grade fluid with variable thermal conductivity has been discussed by Hayat and Abbasi [9]. Slip effects on the peristaltic motion of third grade fluid in a planar channel have been addressed by Hayat and Mehmood [10]. Analytic solutions for heat and mass transfer in the peristaltic flow of fourth grade fluid have been obtained by Hayat and Noreen [11]. elmaboud *et al.* [12] discussed the heat transfer effects in the peristaltic fow of couple stress fluid. Heat transfer in the unsteady pulsatile flow through an annulus is studied by elmaboud and Mekheimer [13]. Combined effects of slip and heat transfer on the peristaltic flow of fourth grade fluid in an inclined channel are addressed by Mehmood *et al.* [14]. Peristaltic flow of third grade fluid in a complaint walls channel is investigated by Hina *et al.* [15]. Recently Mekheimer and El Kot [16] provided the mathematical modeling for unsteady flow of Sisko fluid through anisotropically tapered elastic arteries.

Heat transfer in the peristaltic flows is involved in many complicated processes in tissues such as heat conduction in tissues, heat convection due to the blood flow through the pores of the tissues and radiation between surface and its environment. Mixed convective peristaltic flow of viscous fluid past a vertical wavy surface has been studied by Jang and Yan [17]. Eldabe *et al.* [18] addressed the peristaltic flow of third grade fluid with temperature dependent viscosity. Influence of heat on the peristaltic flow of viscous fluid filling a porous space is analyzed

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by Kothandapani and Srinivas [19]. Mekheimer and elmaboud [20] studied the effects of heat transfer on the hydro-magnetic peristaltic flow in a vertical annulus. Srinivas and Kothandapani [21] analyzed the effect of wall properties and heat and mass transfer on the magneto-hydrodynamics peristaltic flow through a porous space. Muthuraj and Srinivas [22] analyzed the mixed convective heat and mass transfer in a vertical wavy channel with travelling waves and porous medium. Srinivas *et al.* [23] discussed the mixed convective peristaltic flow of viscous fluid in an asymmetric channel. Srinivas and Muthuraj [24] investigated combined effects of chemical reaction and space porosity on hydro-magnetic mixed convective peristaltic flow in a vertical asymmetric channel. Hayat *et al.* [25] discussed the simultaneous effects of heat and mass transfer on peristaltic motion of second grade fluid with wall properties.

Energy flux caused by concentration gradient was first discovered experimentally by Dufour and is called thermal-diffusion (Dufour) effect. On the other hand mass flux can also be developed by temperature gradient and is termed as diffusion-thermo (Soret) effect. Although the effects of thermal-diffusion and diffusion-thermo are considered small order of magnitude in comparison to the effects described by Fourier's or Fick's law but there are situations where such effects cannot be ignored. For instance, the thermal diffusion (Dufour) effect is employed for isotope separation and in mixtures between gases with very light molecular weight (H_2, H_e) and of medium molecular weight (N_2, air), the diffusion-thermo (Soret) effect cannot be neglected. It is revealed that mixed convection in the peristaltic flow of fourth grade fluid is not yet available in the literature. Thus present work reports the nonlinear mixed convective peristaltic flow of fourth grade fluid under viscous dissipation, Dufour and Soret effects. The presence of these effects lead to a coupled nonlinear boundary value problem (BVP) which is solved numerically by an implicit finite difference scheme known as Keller–box method [26,27]. Graphs showing the pressure rise, temperature, concentration and heat transfer coefficient for several values of parameters have been presented and discussed in detail.

2. Problem formulation

Let us consider the flow of an incompressible fourth grade fluid in a symmetric channel of uniform thickness $2d_1$. The shape of propagated waves is of the forms

$$Y = \pm\overline{\eta}(x,t) = \pm\left[d_1 + a\sin\frac{2\pi}{\lambda}(X - ct)\right], \tag{1}$$

in which c is the wave speed and a and λ are the wave amplitude and wavelength respectively. The relevant equations governing for the flow analysis under consideration are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{2}$$

$$\rho\left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right]U = -\frac{\partial P}{\partial X} + \frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y} + \rho g\alpha(T - T_0) + \rho g\beta_c(C - C_0), \tag{3}$$

$$\rho\left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right]V = -\frac{\partial P}{\partial Y} + \frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y}, \tag{4}$$

$$\rho C_p\left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right]T = \kappa\left[\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right] + \frac{\rho D K_T}{C_s}\times\left[\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right] \tag{5}$$

$$+ S_{XX}\frac{\partial U}{\partial X} + S_{YY}\frac{\partial V}{\partial Y} + S_{XY}\left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right), \tag{6}$$

$$\left[\frac{\partial}{\partial t} + U\frac{\partial}{\partial X} + V\frac{\partial}{\partial Y}\right]C = D\left[\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right] + \frac{D K_T}{T_m}\left[\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right], \tag{7}$$

where the constitutive relation for an extra stress tensor in a fourth grade fluid is

$$\begin{aligned}\mathbf{S} \;=\; & \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 + \beta_1\mathbf{A}_3 + \beta_2(\mathbf{A}_1\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1) + \beta_3(tr\mathbf{A}_1^2)\mathbf{A}_1 \\ & \gamma_1\mathbf{A}_4 + \gamma_2(\mathbf{A}_1\mathbf{A}_3 + \mathbf{A}_3\mathbf{A}_1) + \gamma_3\mathbf{A}_2^2 + \gamma_4(\mathbf{A}_1^2\mathbf{A}_2 + \mathbf{A}_2\mathbf{A}_1^2) + \\ & \gamma_5(tr\mathbf{A}_2)\mathbf{A}_2 + \gamma_6(tr\mathbf{A}_2)\mathbf{A}_1^2 + (\gamma_7 tr\mathbf{A}_3 + \gamma_8 tr(\mathbf{A}_2\mathbf{A}_1))\mathbf{A}_1,\end{aligned}$$

with the Rivlin–Ericksen tensors prescribed as

$$\mathbf{A}_1 = \text{grad}\mathbf{V} + (\text{grad}\mathbf{V})^T,$$
$$\mathbf{A}_n = \frac{d\mathbf{A}_{n-1}}{dt} + \mathbf{A}_{n-1}(\text{grad}\mathbf{V}) + (\text{grad}\mathbf{V})^T\mathbf{A}_{n-1} \quad \text{for} \;\; n > 1.$$

where T_1 and C_1, T_0 and C_0 indicate the temperature and the mass concentration at the upper and lower walls of channel respectively, u, v the velocities in x- and y-directions, p the pressure, κ the thermal conductivity, σ the electrical conductivity of fluid, C_p the specific heat at constant volume, T the temperature of fluid, C the mass concentration, D the coefficient of mass diffusivity, K_T the thermal diffusion ratio, T_m the mean temperature, ρ the density, α the coefficient of thermal expansion, β_c the coefficient of expansion with concentration, g the acceleration due to gravity, C_s the concentration susceptibility, $\alpha_i(i = 1, 2), \beta_i(i = 1 - 3)$ and $\gamma_i(i = 1 - 8)$ are the material constants in the fourth grade fluid and S_{xx}, S_{xy} and S_{yy} are the components of an extra stress tensor $\mathbf{S}$. Eqs. (3) and (4) yield the following compatibility relation

$$\frac{d}{dt}\left[\frac{\partial U}{\partial Y} - \frac{\partial V}{\partial X}\right] = \frac{\partial}{\partial Y}\left[\frac{\partial S_{XX}}{\partial X} + \frac{\partial S_{XY}}{\partial Y}\right] - \frac{\partial}{\partial X}\left[\frac{\partial S_{XY}}{\partial X} + \frac{\partial S_{YY}}{\partial Y}\right] + \rho g\alpha\frac{\partial T}{\partial Y} + \rho g\beta_c\frac{\partial C}{\partial Y}. \tag{8}$$

If (x, y) and (u, v) are the coordinates and velocity components in the wave frame then

$$x = X - ct,\; y = Y,\; u = U - c,\; v = V, \quad p(x) = P(X, t), \tag{9}$$

$\psi(x, y, t)$ is the stream function then writing

$$u = \frac{\partial\psi}{\partial y},\; v = -\frac{\partial\psi}{\partial x}$$

and employing the definitions of following dimensionless variables

$$\begin{aligned}\psi^* \;=\; & \frac{\psi}{cd_1}, \quad x^* = \frac{x}{\lambda}, \quad y^* = \frac{y}{d_1}, \quad t^* = \frac{ct}{\lambda}, \quad \eta^* = \frac{\eta}{d_1}, \\ & p^* = \frac{d_1^2 p}{c\lambda\mu}, \quad S_{ij}^* = \frac{dS_{ij}}{c\mu}, \\ \alpha_i^* \;=\; & \frac{\alpha_i c}{\mu d_1}, \quad \beta_i^* = \frac{\beta_i c^2}{\mu d_1^2}, \quad \gamma_i^* = \frac{\gamma_i c^3}{\mu d_1^3}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \\ & \phi = \frac{C - C_0}{C_1 - C_0}\end{aligned}$$