Journal of the Taiwan Institute of Chemical Engineers 45 (2014) 2163–2169

Contents lists available at ScienceDirect

Journal of the Taiwan Institute of Chemical Engineers

journal homepage: www.elsevier.com/locate/jtice

Mixed convection flow of a micropolar fluid over a continuously moving vertical surface immersed in a thermally and solutally stratified medium with chemical reaction

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ARTICLE INFO

Article history:
Received 29 January 2014
Received in revised form 6 July 2014
Accepted 12 July 2014
Available online 10 August 2014

Keywords:
Heat and mass transfer
Micropolar fluid
Thermal/soutal stratification
Mixed convection

ABSTRACT

The coupled heat and mass transfer by mixed convection boundary layer flow of a micropolar fluid over a continuously moving isothermal vertical surface immersed in a thermally and solutally stratified medium have been investigated in the presence of chemical reaction effect. The governing partial differential equations are transformed into a set of non-similar equations and solved numerically by the Keller box method. Comparison with previously published work are performed and the results are found to be in excellent agreement. A representative set of numerical results for the skin-friction coefficient as well as the Nusselt number and the Sherwood number is presented graphically for various parametric conditions and discussed.

1. Introduction

In recent years, the dynamics of micropolar fluids, originated from the theory of Eringen [1,2] has been a popular area of search. This theory takes into account the effect of local rotary inertia and couple stresses arising from practical micro-rotation. The theory is expected to provide a mathematical model for the non-Newtonian fluid behavior observed in certain man-made liquids such as polymers, colloidal suspensions, fluids with additives, suspension solutions, and animal blood, *etc*. This theory also capable of explaining the experimentally observed phenomena of drag reduction near a rigid body in fluids containing small amount of additives when compared with the skin friction in the same fluids without additives. An excellent review about micropolar fluid mechanics was provided by Ariman et al. [3,4].

Mansour et al. [5] discussed heat and mass transfer in on magnetohydrodynamic flow of micropolar fluids in a circular cylinder with uniform heat and mass flux. Siddheshwar and Manjunath [6] presented numerical study of unsteady convective diffusion with heterogeneous chemical reaction in a plane-Poiseuille flow of a micropolar fluid. EL-Kabeir [7] analyzed the radiation effect on forced convection flows in micropolar fluids with variable viscosity. The coupled heat and mass transfer by free convection flow over a cone with uniform suction or injection in micropolar fluid have been considered by EL-Kabeir et al. [8]. Modther et al. [9] studied the effects of mixed thermal boundary condition and magnetic field on free convection flow about a cone in micropolar fluids. Modather et al. [10] have also considered the effect of chemical reaction on the heat and mass transfer of micropolar fluids in a saturated porous medium over an infinite moving permeable plate in presence of magnetic field. Magyari and Chamkha [11] studied the combined effect of heat generation or absorption and first-order chemical reaction on micropolar fluid flows over a uniformly stretched permeable surface. EL-Kabeir et al. [12] discussed the problem of heat transfer in a micropolar fluid flow past a permeable continuous moving surface. Rashidi et al. [13] have obtained analytic approximate solutions for heat transfer of a micropolar fluid through a porous medium with radiation effect. Pal [14] analyzed the combined effects of non-uniform heat source/sink and thermal radiation on heat transfer over an unsteady stretching permeable surface. Chamkha et al. [15] studied the problem of unsteady MHD natural convection from a heated vertical porous plate in a micropolar fluid with Joule heating, chemical reaction and radiation effects. Narayana et al. [16] have studied the effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system. Hareesh and Narayana [17] presented an analysis of heat and mass transfer in MHD micropolar flow over a vertical moving porous plate with

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http://dx.doi.org/10.1016/j.jtice.2014.07.002

Nomenclature

a, b	constants
B	spin gradient viscosity parameter
c_p	specific heat at constant pressure
C_f	local skin-friction coefficient
D	mass diffusivity
f	dimensionless stream function
g^*	gravitational acceleration
g	dimensionless microrotation
Gr	Grashof number, $g\beta_T(T_w - T_{\infty,0})L^3/\nu^2$
j	microinertia density
R	material parameter
K_1	dimension of chemical reaction
L	characteristic length
N	component of the microrotation vector normal to x–y plane
Nu	Nusselt number
Pr	Prandtl number, ν/α
Re	Reynolds number UL/ν
S_1, S_2	Thermal and solutal stratification parameters
Sc	Schmidt number, ν/D
Sh	Sherwood number
T	temperature
u	velocity component in the x-direction
U	velocity of the moving surface
v	velocity component in the y-direction
x	vertical coordinate
y	horzental coordinate

Greek Symbols

α	thermal diffusivity
β_c	coefficient of concentration expansion
β_T	coefficient of thermal expansion
γ	dimensionless of chemical reaction
γ^*	spin-gradient viscosity
η	pseudo-similarity variable
λ	(=constant) is the mixed convection parameter Gr/Re^2
Λ	concentration to thermal buoyancy ratio, $\beta_c(C_w - C_\infty)/(\beta_T(T_w - T_\infty))$
ϕ	dimensionless concentration, $(C - C_\infty)/(C_w - C_\infty)$
θ	dimensionless temperature, $(T - T_\infty)/(T_w - T_\infty)$
ξ	dimensionless streamwise coordinate
ν	kinematic viscosity
ρ	density
ψ	stream function

Subscripts

w	condition at the wall
∞	condition at infinity

radiation absorption effect. Chamkha et al. [18] investigated the effect of chemical reaction on heat and mass transfer by MHD natural convection of micropolar fluid about a radiate truncated cone. The problem of heat and mass transfer in on MHD micropolar fluid in rotating frame of reference in the presence of thermal radiation and chemical reaction effects is considered by Narayana et al. [19]. Chamkha et al. [20] have recently analyzed the effects of chemical reaction on unsteady coupled heat and mass transfer by mixed convection flow of a micropolar fluid near the stagnation point on a vertical radiate surface.

On other hand, the analysis of free or mixed convection in a thermally and solutally stratified medium is a fundamentally interesting and important problem because of the broad range of engineering applications. They include heat rejection into the environment such as lakes, rivers and the seas; thermal energy storage systems such as solar ponds; and heat transfer from thermal sources such as the condensers of power plants. Nakayama and Koyama [21,22] studied free convection over a vertical flat plate embedded in a thermally stratified porous medium. Chamkha [23] has analyzed hydromagnetic natural convection from an isothermal inclined surface adjacent to a thermally stratified porous medium. Takhar et al. [24] have studied the natural convection boundary layer over a continuously moving isothermal vertical surface immersed in a thermally stratified medium. Murthy et al. [25] have analyzed the effect of double stratification on double diffusive natural convection from a vertical impermeable flat plate in porous media. Narayana and Murthy [26] have studied the heat and mass transfer by natural convection from a vertical surface embedded in a doubly stratified porous medium. Beg et al. [27] investigated the free convection flow boundary layer flow over a continuously moving plate immersed in a thermally-stratified high porosity porous medium. The problem of coupled heat and mass transfer by natural convection flow from a vertical wavy surface in a fluid saturated porous medium with thermal and mass stratification has been considered by Cheng [28]. Muhaimin et al. [29] investigated the effects of variable viscosities and thermal stratification on the MHD mixed convective heat and mass transfer of an electrically conducting fluid past a porous wedge. The problem of natural convection heat and mass transfer along a vertical plate embedded in a doubly stratified micropolar fluid saturated non-Darcy porous medium is presented by Srinivasacharya and Reddy [30]. Rashad et al. [31] presented a numerical investigation of non-Darcy natural convection from a vertical cylinder embedded in a thermally stratified and nanofluid-saturated porous media.

In the present work, the problem of coupled heat and mass transfer by mixed convection boundary layer flow of a micropolar fluid over a continuously moving isothermal vertical surface immersed in a thermally and solutally stratified medium is considered. The governing boundary-layer equations have been transformed to a non-similar form, and these have been solved numerically by an implicit finite difference scheme known as the Keller box method. The effects of thermal and solutal stratification parameters and micropolar parameter on the skin-friction coefficient as well as the Nusselt number and the Sherwood number have been shown graphically and discussed.

2. Analysis

Consider steady, laminar, heat and mass transfer by mixed convection boundary layer flow of a micropolar fluid over a continuously moving isothermal vertical surface immersed in a thermally and solutally stratified medium in the presence of chemical reaction effect. The flow configuration is shown schematically in Fig. 1 together with the corresponding Cartesian coordinates in the vertical and horizontal directions. The flat surface is maintained at a uniform temperature T_w and a uniform concentration C_w moving with a constant velocity U in a x-direction, in a stable thermally and solutally ambient fluid. The ambient medium is assumed to be vertically linearly stratified with respect to both temperature and concentration in the form $T_\infty(x) = T_{\infty,0} + ax$ and $C_\infty(x) = C_{\infty,0} + bx$, respectively, where a and b are constants and varied to alter the intensity of stratification in the