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PII: S0045-7825(18)30334-7
DOI: https://doi.org/10.1016/j.cma.2018.07.003
Reference: CMA 11975

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date : 15 June 2017
Revised date : 14 May 2018
Accepted date : 2 July 2018

Please cite this article as: C. Rodrigo, X. Hu, P. Ohm, J.H. Adler, F.J. Gaspar, L.T. Zikatanov, New stabilized discretizations for poroelasticity and the Stokes' equations, *Comput. Methods Appl. Mech. Engrg.* (2018), https://doi.org/10.1016/j.cma.2018.07.003

New stabilized discretizations for poroelasticity and the Stokes' equations

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Abstract

In this work, we consider the popular P1-RT0-P0 discretization of the three-field formulation of Biot's consolidation problem. Since this finite-element formulation is not uniformly stable with respect to the physical parameters, several issues arise in numerical simulations. For example, when the permeability is small with respect to the mesh size, volumetric locking may occur. To alleviate such problems, we consider a well-known stabilization technique with face bubble functions. We then design a perturbation of the bilinear form, which allows for local elimination of the bubble functions. We further prove that such perturbation is consistent and the resulting scheme has optimal approximation properties for both Biot's model as well as the Stokes' equations. For the former, the number of degrees of freedom is the same as for the classical P1-RT0-P0 discretization and for the latter (Stokes' equations) the number of degrees of freedom is the same as for a P1-P0 discretization. We present numerical tests confirming the theoretical results for the poroelastic and the Stokes' test problems.

Keywords: Stable finite elements, poroelasticity, Stokes' equations

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1. Introduction

The interaction between the deformation and fluid flow in a fluid-saturated porous medium is the object of study in poroelasticity theory. Such coupling has been modelled in the early one-dimensional work of Terzaghi [1]. A more general three-dimensional mathematical formulation was then established by Maurice Biot in several pioneering publications (see [2] and [3]). Biot's models are widely used nowadays in the modeling of many applications in different fields, ranging from geomechanics and petroleum engineering, to biomechanics. The existence and uniqueness of the solution for these problems have been investigated by Showalter in [4] and by Zenisek in [5]. Regarding the numerical simulation of the poroelasticity equations, there have been numerous contributions using finite-difference schemes [6, 7] and finite-volume methods (see [8, 9] for recent developments). Finite-element methods, which are the subject of this work, have also been considered (see for example the monograph by Lewis and Schrefler [10] and the references therein).

Stable finite-element schemes are constructed by either choosing discrete spaces satisfying appropriate inf-sup (or LBB) conditions, or applying suitable stabilization techniques to unstable

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Preprint submitted to Computer Methods in Applied Mechanics and Engineering *July 8, 2018*