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Blended B-Spline Construction on Unstructured Quadrilateral and Hexahedral Meshes with Optimal Convergence Rates in Isogeometric Analysis

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Abstract

We present a novel blended B-spline method to construct bicubic/tricubic splines over unstructured quadrilateral and hexahedral meshes for isogeometric analysis. C^1 and (truncated) C^2 B-spline functions are used in regular elements, whereas C^0 and (truncated) C^1 B-spline functions are adopted in boundary elements and interior irregular elements around extraordinary edges/vertices. The truncation mechanism is employed for a seamless transition from irregular to regular elements. The resulting smoothness of the blended construction is C^2-continuous everywhere except C^0-continuous around extraordinary edges and C^1-continuous across the interface between irregular and regular elements. The blended B-spline construction yields consistent parameterization during refinement and exhibits optimal convergence rates. Spline functions in the blended construction form a non-negative partition of unity, are linearly independent, and support Bézier extraction such that the construction can be used in existing finite element frameworks. Several examples provide numerical evidence of optimal convergence rates.

Keywords: Blended B-Spline Construction, Extraordinary Vertices/Edges, Optimal Convergence Rates, Unstructured Quadrilateral and Hexahedral Meshes, Mixed Smoothness, Isogeometric Analysis.

1. Introduction

Isogeometric analysis (IGA) was introduced to bridge the gap between computer-aided design (CAD) and traditional finite element analysis (FEA) by utilizing the same basis of a CAD representation in analysis [16, 10]. In addition to CAD representations, unstructured quadrilateral (quad) and hexahedral (hex) meshes can also serve as important input control meshes[1] for IGA. For instance, many techniques have been developed to convert imaging data to such meshes [45]. Unstructured quad/hex meshes inevitably involve extraordinary vertices/edges. In the interior of a quad/hex mesh, an *extraordinary vertex/edge* is a vertex/edge shared by other than four quad/hex elements, respectively. Endpoints of extraordinary edges in 3D are also extraordinary vertices. Generally, in a hex mesh, there exist 3D extraordinary vertices that cannot be obtained by sweeping 2D counterparts. How to deal with extraordinary vertices/edges is the key to employing unstructured quad/hex meshes in IGA, and developing a spline basis with desired properties, such as non-negative partition of unity, linear independence, smoothness (preferably G^1 or better), nested

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[1] The term "control mesh" implies the odd degree case, where each control point corresponds to a mesh vertex.