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PII: S0045-7825(18)30359-1
DOI: https://doi.org/10.1016/j.cma.2018.07.026
Reference: CMA 11998

To appear in: *Comput. Methods Appl. Mech. Engrg.*

Received date : 11 October 2017
Revised date : 19 July 2018
Accepted date : 19 July 2018

A Scalable Solution Strategy for High-Order Stabilized Finite-Element Solvers using an Implicit Line Preconditioner

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Abstract

This paper presents a robust, efficient, and strongly scalable solution methodology for simulation of complex turbulent flows on unstructured grids. The compressible Reynolds averaged Navier-Stokes (RANS) equations and the negative Spalart-Almaras (SA) turbulence model are discretized, in coupled form, using a Streamline Upwind Petrov-Galerkin (SUPG) scheme. The time integration is fully implicit, and the discretized equations are advanced toward a steady-state solution using a pseudo-transient continuation (PTC). For solution of the linearized systems, a preconditioned Krylov solver is used. Seeking robustness, Krylov solvers are commonly preconditioned using incomplete factorization methods such as ILU(k). However, these methods are neither memory efficient, nor strongly scalable. To provide a better alternative, the implicit line solution method, which has been traditionally used in finite-volume methods, is revised and enhanced to solve stiffer linear systems. In the developed method, the lines are generated using a matrix-based approach, which connects the strongly-coupled unknowns. In addition, to improve the robustness of the line solver for high-CFL systems, a dual-CFL strategy, with a lower CFL number in the preconditioner matrix, is developed. Also, it is shown that for high-order continuous finite-element discretizations, the interconnections of the degrees of freedom on a line form a banded matrix which is wider than tridiagonal, but still can be factorized completely without generating any fill-ins. The developed line preconditioner is strongly scalable and, in contrast to the ILU factorization, its convergence behavior does not depend on the number of partitions. Two three-dimensional numerical examples are presented in which the performance of the line preconditioner is compared with that of the ILU(k) preconditioner. This comparison shows that, in addition to robustness improvements, the line preconditioner offers significant benefits in terms of memory efficiency.

1. Introduction

During the last few decades, there has been a growing interest in the development and utilization of stabilized finite-element (FE) methods for the solution of the compressible Reynolds-Averaged Navier-Stokes (RANS) equations on unstructured grids. The major characteristics of these methods are the continuity of the solution space and minimal cross-wind dissipation. The latter is obtained by a stabilization term which can be interpreted as a perturbation to the classical Galerkin weight functions, such that the up-winding effect is made in the characteristic directions; these methods are also known as Petrov-Galerkin (PG) methods. In the field of computational fluid dynamics (CFD), PG methods are naturally compared with the traditional second-order finite-volume (FV) as well as the developing discontinuous Galerkin (DG) finite-element methods. A major advantage of the PG methods over the FV methods is the use of nearest neighbor stencils to reach high-order discretizations. The use of nearest neighbor stencils also facilitates the accurate linearization of the nonlinear residual. This provides significant benefits in development of Newton-based nonlinear solvers and for sensitivity analysis [1]. Compared to DG schemes, several studies have shown that for moderate discretization orders, and for comparable accuracies, PG schemes require significantly less computational resources than DG schemes [2, 3]. Also, for viscous flows, DG schemes require super-parametric elements (i.e. elements in which the geometry is defined by shape functions of order of higher than that of the field variables) to deliver the nominal order of accuracy of the discretization [4, 5], whereas PG schemes can properly work with iso-parametric elements (i.e. elements in which the shape functions of the geometrical and field variables have the same order). Therefore, PG schemes do not require curved meshes for second-order-accurate

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