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Abstract

We derive a posteriori error estimators for an optimal control problem governed by a convection–reaction–diffusion equation; control constraints are also considered. We consider a family of low–order stabilized finite element methods to approximate the solutions to the state and adjoint equations. We obtain a fully computable a posteriori error estimator for the optimal control problem. All the constants that appear in the upper bound for the error are fully specified. Therefore, the proposed estimator can be used as a stopping criterion in adaptive algorithms. We also obtain a robust a posteriori error estimator for when the error is measured in a norm that involves the dual norm of the convective derivative. Numerical examples, in two and three dimensions, are presented to illustrate the theory.

Keywords: linear–quadratic optimal control problem; convection–reaction–diffusion equation; stabilized methods; fully computable a posteriori error estimator; robust a posteriori error estimator.

1. Introduction

The purpose of this work is to construct and analyze a posteriori error estimators for a control–constrained optimal control problem involving a convection–reaction–diffusion equation as state equation. To describe our problem, let Ω be an open and bounded polytopal domain in $\mathbb{R}^d$, $d \in \{2, 3\}$, with Lipschitz boundary $\partial\Omega$. Given a desired state $\mathsf{y}_\Omega \in L^2(\Omega)$, we define the cost functional

$$J(\mathsf{y}, \mathsf{u}) = \frac{1}{2}\|\mathsf{y} - \mathsf{y}_\Omega\|^2_{L^2(\Omega)} + \frac{\vartheta}{2}\|\mathsf{u}\|^2_{L^2(\Omega)}, \qquad (1)$$

where $\vartheta > 0$ denotes the so–called regularization parameter. We shall be concerned with the following optimal control problem: Find

$$\min\ J(\mathsf{y}, \mathsf{u}) \qquad (2)$$

subject to the convection–reaction–diffusion equation

$$-\nu\Delta\mathsf{y} + \mathbf{b}\cdot\nabla\mathsf{y} + \kappa\mathsf{y} = \mathsf{f} + \mathsf{u} \quad \text{in } \Omega, \qquad \mathsf{y} = 0 \quad \text{on } \partial\Omega, \qquad (3)$$

and the control constraints

$$\mathsf{u} \in \mathbb{U}_{ad}, \qquad \mathbb{U}_{ad} := \{\mathsf{v} \in L^2(\Omega) : \ \mathsf{a} \le \mathsf{v}(\boldsymbol{x}) \le \mathsf{b} \text{ for almost every } \boldsymbol{x} \text{ in } \Omega\}. \qquad (4)$$

Here, the bounds $\mathsf{a}, \mathsf{b} \in \mathbb{R}$ and are such that $\mathsf{a} < \mathsf{b}$ and $\mathsf{f} \in L^2(\Omega)$. Assumptions on ν, $\mathbf{b}$ and κ are deferred until later.