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BERNSTEIN-BÉZIER BASES FOR TETRAHEDRAL FINITE ELEMENTS

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ABSTRACT. We present a new set of basis functions for $H(\text{curl})$-conforming, $H(\text{div})$-conforming, and L^2-conforming finite elements of arbitrary order on a tetrahedron. The basis functions are expressed in terms of Bernstein polynomials and augment the natural H^1-conforming Bernstein basis. The basis functions respect the differential operators, namely, the gradients of the high-order H^1-conforming Bernstein-Bézier basis functions form part of the $H(\text{curl})$-conforming basis, and the curl of the high-order, non-gradients $H(\text{curl})$-conforming basis functions form part of the $H(\text{div})$-conforming basis, and the divergence of the high-order, non-curl $H(\text{div})$-conforming basis functions form part of the L^2-conforming basis.

Procedures are given for the efficient computation of the mass and stiffness matrices with these basis functions without using quadrature rules for (piecewise) constant coefficients on affine tetrahedra. Numerical results are presented to illustrate the use of the basis to approximate representative problems.

1. INTRODUCTION

Let $\mathbb{P}_n$ be the space of polynomials of degree no greater than n on the tetrahedron, $\widetilde{\mathbb{P}}_n$ be the space of homogeneous polynomials of degree n, and $\mathbb{P}_n^3 := [\mathbb{P}_n, \mathbb{P}_n, \mathbb{P}_n]^T$ is the vector space whose components lie in $\mathbb{P}_n$. We present a set of Bernstein-Bézier bases for the following two families of arbitrary order polynomial exact sequences on a tetrahedron T:

TABLE 1. Two exact sequences on a tetrahedron T

	$H^1(T)$		$H(\text{curl}; T)$		$H(\text{div}; T)$		$L^2(T)$
First sequence:	$\mathbb{P}_{n+1}$	$\xrightarrow{\nabla}$	$\mathbb{P}_n^3 \oplus \boldsymbol{x} \times \widetilde{\mathbb{P}}_n^3$	$\xrightarrow{\nabla\times}$	$\mathbb{P}_n^3 \oplus \boldsymbol{x}\widetilde{\mathbb{P}}_n$	$\xrightarrow{\nabla\cdot}$	$\mathbb{P}_n$
Second sequence:	$\mathbb{P}_{n+2}$	$\xrightarrow{\nabla}$	$\mathbb{P}_{n+1}^3$	$\xrightarrow{\nabla\times}$	$\mathbb{P}_n^3$	$\xrightarrow{\nabla\cdot}$	$\mathbb{P}_{n-1}$

The bases for the H^1-case is taken to be the standard Bernstein polynomials as in [2]. The basis for the $H(\text{curl})$, $H(\text{div})$ and L^2 cases are also constructed in terms of Bernstein polynomials in such a way that: the gradients of the high-order H^1-conforming Bernstein-Bézier basis functions are part of the $H(\text{curl})$-conforming basis; the curl of the high-order, non-gradient $H(\text{curl})$-conforming basis functions are part of the $H(\text{div})$-conforming basis; and, the divergence of the high-order, non-curl $H(\text{div})$-conforming basis functions are part of the L^2-conforming basis.

The construction of high-order finite element basis functions for the polynomial exact sequences has been extensively studied in the literature, see the hierarchical bases in [4, 8, 14], and the Bernstein-Bézier bases in [2, 3, 7]. Previous work [2]

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