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Strain-gradient vs damage-gradient regularizations of softening damage models

Duc Trung Le[a], Jean-Jacques Marigo[b], Corrado Maurini[a], Stefano Vidoli[c,*]

[a] *Sorbonne Université, Centre National de la Recherche Scientifique, UMR 7190, Institut Jean Le Rond d'Alembert, F-75005 Paris, France*
[b] *Laboratoire de Mécanique des Solides, Ecole Polytechnique, Palaiseau, France*
[c] *Dipartimento di Ingegneria Strutturale e Geotecnica, Università di Roma La Sapienza, Rome, Italy*

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Abstract

Local damage models with softening needs localization limiters to preserve the mathematical and physical consistency. In this paper we compare the properties of strain-gradient and damage-gradient regularizations. Gradient-damage models introduce a quadratic dependency of the dissipated energy on the gradient of the damage field and are nowadays extensively used as phase-field approximation of brittle fracture. Their key feature is to provide a smeared approximation of a crack as a band of localized damage with a finite energy dissipation per unit of surface, that can be identified with the fracture toughness of the Griffith model. Strain gradient models introduce a quadratic dependence of the elastic energy on the gradient of the strain field. A similar term can be physically interpreted as the presence in the material of linear, but nonlocal, stiffnesses, that can be eventually be affected by damage. Despite this attractive interpretation, we have found that strain-gradient regularized models can hardly be used to approximate brittle fracture, because smeared cracks with non-vanishing and finite dissipated energies are hardly obtained. Our analysis is based on variational models and focuses on the one-dimensional traction problem.

Keywords: Fracture mechanics; Softening materials; Damage; Phase-field; Regularization; Strain-gradient

1. Introduction

Damage models describe material failure by an additional internal variable modulating the elastic stiffness and inducing an internal energy dissipation in the material. In order to model material failure observed in quasi-brittle materials, damage models must include stress-softening, *i.e.* the reduction of the admissible stress domain for increasing damage. It can be shown that this constitutive property is associated to the loss of the uniqueness of the solution and the appearance of localized states, a key aspect of the behavior of real structures. It is widely recognized that in order to preserve the mathematical consistency and the capability of formulating mesh-independent numerical approximation, strain-softening damage models must be regularized by introducing some sort of non-local effects, [1].

* Corresponding author.
E-mail address: stefano.vidoli@uniroma1.it (S. Vidoli).

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Among the several regularization strategies proposed in the literature, we may first distinguish between those based on the introduction of smoothed variables in the form of convolution integrals of the local variables (*nonlocal damage models*, see e.g. [2]) and those penalizing extreme localizations through gradient terms (*gradient damage models*). One can further classify the possible approaches between those introducing the regularization through nonlocal terms in the damage variable (*damage-regularized damage models*, see e.g. [3]) or in the elastic strain variable (*strain-regularized damage models*, see e.g. [4–8]). In [9] the authors perform a throughout analysis of the possible nonlocal regularization strategies. They conclude that only a very small subset of the models proposed in the literature are really effective in providing mathematically well-posed and thermodynamically consistent problems. They suggest the use of gradient approaches. In the present paper, we will focus on gradient-type regularizations and compare the properties of *strain-gradient* (SG) and *damage-gradient* (DG) models, discussing their appropriateness to model fracture of brittle solids.

Failure often manifests in the form of cracks, *i.e.* surfaces of discontinuity of the displacement field, where the material "breaks". Damage models are often regarded as *smeared* crack models smoothing out this discontinuity in bands of finite length. They are opposed to *discrete* approaches, which models the sharp discontinuities and the possible jumps of the displacement field explicitly. The most widely accepted *discrete* crack model is the *Griffith* model, which assumes that the creation of the crack is done at expenses of an energy dissipation proportional to the crack surface, and that the crack propagation is not possible if this energy is larger than the elastic energy rewarded during a virtual crack propagation. In the end of the nineties, this energetic theory has been put in a precise mathematical setting by [10], allowing for the generalization to the case of cracks of arbitrary shapes and with complex evolution in time. The key advantage of the Griffith model with respect to the damage model is its simplicity when the crack path is postulated in advance. Vice-versa, damage models are much more practical when considering cracks of unknown and possibly complex shapes, because they do not require the explicit description of the crack geometry: the damage field is treated as a standard field modulating the stiffness and the cracks are identified *a posteriori* as the regions where the elastic stiffness vanishes. Another fundamental advantage of damage models is to be able to retrieve crack nucleation, *i.e.* the creation of a crack from an intact material with smooth boundaries.

The so-called phase-field models of fracture are gaining an increasing popularity because they combine the advantages of the smeared and the discrete approaches and give a precise meaning to the idea of using damage models to approximate sharp cracks, or vice-versa.[1] These models have been independently developed in different contexts. In applied mathematics and theoretical mechanics, they arise through the transposition [11] of the regularized models for image segmentation [12] to the variational formulation of fracture mechanics [10]. In physics they derive from the application to fracture [13,14] of the Ginzburg–Landau theories of phase transformations. Phase-field models of fracture are also a special type of the DG models presented in [3].

Previous studies [15,16] analyzed in depth the properties of DG models, showing that they can be regarded as a regularized version of the variational theory of brittle fracture, where the regularization parameter, the internal length, can be set to recover crack nucleation in agreement with experimental observations [17]. In this paper we analyze whether a strain-gradient (SG) model can be similarly used to approximate brittle fracture *à la* Griffith. In particular, is it possible to recover with SG models the energetic equivalence with brittle fracture?

Our analysis relies on a variational approach and focuses only on a one-dimensional traction problem. Specifically we consider strain-gradient damage model where the elastic energy density is quadratic in both the strain and the strain-gradient. The corresponding local $\mathtt{E}(\alpha)$ and nonlocal $\mathtt{G}(\alpha)$ stiffnesses are assumed to depend on the damage variable α with the local stiffness vanishing for $\alpha = 1$. In a one dimensional traction problem, we identify cracks as solutions with vanishing stress and a smeared displacement jump. We show that two fundamentally different qualitative behaviors are possible, depending on whether the limit value of the nonlocal stiffness $\mathtt{G}(\alpha = 1)$ is vanishing or not:

1. when $\mathtt{G}(1) = 0$ the regularizing term is not effective and the SG-model shows pathologies similar to the non regularized case, *e.g.* the creation of cracks with vanishing energy dissipation, resulting in mesh-dependence of numerical approximations;
2. when $\mathtt{G}(1) > 0$ cracks, the creation of cracks requires to fully damage the whole bar and, hence a dissipation proportional to the bar length.

[1] If one regards the phase-field model as a damage model with an internal length, the damage model is richer than the Griffith model and one should regard the damage model as the true physical model and the Griffith model as an approximation. This view is opposite to that of Gamma-Convergence, where the damage model is regarded as a regularized approximation of the Griffith model.