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# A multi-temporal scale model reduction approach for the computation of fatigue damage

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**Abstract**

One of the challenges of fatigue simulation using continuum damage mechanics framework over the years has been reduction of numerical cost while maintaining acceptable accuracy. The extremely high numerical expense is due to the temporal part of the quantities of interest which must reflect the state of a structure that is subjected to exorbitant number of load cycles. A novel attempt here is to present a non-incremental LATIN-PGD framework incorporating temporal model order reduction. LATIN-PGD method is based on separation of spatial and temporal parts of the mechanical variables, thereby allowing for separate treatment of the temporal problem. The internal variables, especially damage, although extraneous to the variable separation, must also be treated in a tactical way to reduce numerical expense. A temporal multi-scale approach is proposed that is based on the idea that the quantities of interest show a slow evolution along the cycles and a rapid evolution within the cycles. This assumption boils down to a finite element like discretisation of the temporal domain using a set of "nodal cycles" defined on the slow time scale. Within them, the quantities of interest must satisfy the global admissibility conditions and constitutive relations with respect to the fast time scale. Thereafter, information of the "nodal cycles" can be interpolated to simulate the behaviour on the whole temporal domain. This numerical strategy is tested on different academic examples and leads to an extreme reduction in numerical expense.

*Keywords:*
Model reduction, Two-time scale, Fatigue, Damage mechanics, (Visco)plasticity, Cyclic loading, LATIN method

## 1. Introduction

Risk of failure due to fatigue, i.e. due to repetitive loading, is an essential factor of failure for most mechanical components and civil structures (see Sobczyk and Spencer, 1992). Continuum damage mechanics, being a branch of the classical continuum mechanics, introduces an extra internal variable, that quantifies in a thermodynamically consistent framework the loss of load carrying capacity of a structure (see Lemaitre and Desmorat, 2005; Lemaitre, 1996). From the first usage of continuum damage mechanics to predict fatigue (see Chaboche and Lesne, 1988), significant developments have taken place for modelling different types of fatigue processes (see Lemaitre and Desmorat, 2005; Lemaitre, 1996). One of the most important developments, proposed first in Ladevèze and Lemaitre (1984), was the idea of micro-defects closure effects, where the difference in damage behaviour during tension and compression was introduced. This idea has later been used extensively to model fluctuating loads (Desmorat et al., 2007, 2006; Desmorat and Cantournet, 2008; Lemaitre et al., 1999) especially for fatigue applications.

Continuum damage mechanics offers a flexible platform for simulating damage evolution for a large variety of applications, however, this leads to computations, which may be extremely costly. More specifically, the numerical challenge for fatigue simulation in a continuum mechanics framework lies in the number of cycles to be computed, for instance, $10^4$ cycles including 100 time steps per cycle lead to one million of time steps. For fatigue computation applied to a large number of cycles, advanced numerical strategies are then required to circumvent the computational cost due to the large temporal domain.

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