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Comput. Methods Appl. Mech. Engrg. 340 (2018) 824–863

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

On the equivalence between the multiplicative hyper-elasto-plasticity and the additive hypo-elasto-plasticity based on the modified kinetic logarithmic stress rate

Yang Jiao, Jacob Fish*

Columbia University, United States

Received 24 January 2018; received in revised form 8 June 2018; accepted 11 June 2018
Available online 22 June 2018

Abstract

In two theorems presented herein, we prove and subsequently demonstrate in several numerical examples involving homogeneous deformation that for isotropic materials, hyper-elasto-plasticity models based on the multiplicative decomposition of the deformation gradient coincide with an additive hypo-elasto-plasticity model (see Section 3.2) that employs the spin tensor based on the modified kinetic logarithmic rate. In the absence of strain-induced anisotropy (characterized by kinematic hardening herein), this objective stress rate coincides with the kinetic logarithmic rate recently developed by Jiao and Fish, 2017. We also show that other well-known additive decomposition models, such as those based on the Jaumann and logarithmic rates, may considerably deviate from the multiplicative model.

Keywords: Plasticity; Large deformation; Objective stress rate; Logarithmic spin; Elastoplasticity

1. Introduction

Plasticity in the finite deformation regime has been attracting considerable attention since it is one of the most common dissipative phenomena that accompany large deformation of solids (such as metals, polymers, soils etc.). The complexity of modeling large strain plasticity lies in the fact that large deformation of these materials usually involves not only irreversible plastic deformation, but also simultaneous reversible elastic deformation. These two processes with contradictory characteristics are closely coupled in an inextricable manner with geometric nonlinearity. Quantifying elastic and plastic portions of the total finite deformation has been a subject of intensive research in both academia and practicing world. One of the common approaches is to split the rate of deformation into elastic and plastic parts in an additive manner [1]. Another widely recognized approach is to consider the deformation gradient as the multiplicative product of elastic and plastic deformation gradients [2–5]. Among other noteworthy elasto-plastic decompositions (that received somewhat lesser attention) are the additive splits of the Green–Lagrange strain

* Corresponding author.
E-mail address: fishj@columbia.edu (J. Fish).

https://doi.org/10.1016/j.cma.2018.06.017

[6–9], the logarithmic strain [10–12] and the Biot strain [13,14]. In particular, it was recently shown that an additive decomposition of the Green–Lagrange strain indeed results from a thermodynamically consistent formulation which is rooted in a Gibbs potential (complementary energy) based on the second Piola–Kirchhoff Stress [15,16].

The multiplicative decomposition of the deformation gradient finds its roots in the slip theory of crystals [17–19]. It has been recently shown that such a kinematic assumption is a natural result of homogenization of crystalline slips [20,21]. Consequently, the elastic and plastic deformation gradients resulting from such a decomposition can be considered as internal state variables and the corresponding elastoplasticity models can be formulated in a thermodynamically consistent manner. Numerous elastoplasticity models have been developed based on the multiplicative decomposition of the deformation gradient (see [22–38] among others). Due to their adoption of hyperelastic relations in representing elastic response, these models are referred herein to as the multiplicative hyper-elasto-plasticity models.

The additive split of the rate of deformation can be regarded as a direct generalization of the additive split of the strain rate in the classical infinitesimal elastoplasticity theory. It is therefore straightforward to extend the well-established infinitesimal theory into the large deformation regime based on this kinematic assumption [39]. Due to the simplicity and clarity of the constitutive equations, the hypo-elasto-plasticity models have been enjoying popularity in both the academic research (see [40–52] among others) and commercial finite element codes. In the large deformation regime, the time derivative of stress in the infinitesimal theory is replaced with an objective stress rate in order to ensure frame-invariance (objectivity) of the constitutive model. This typically results in a hypoelastic relation for the representation of the elastic response. These models are thus referred herein to as the additive hypo-elasto-plasticity models.

In principle, there exist an infinite number of ways to define the objective stress rate for the corresponding constitutive models to be frame-invariant [53,54]. To address the multiplicity issue, an appropriate choice of the objective stress rates is required to satisfy certain consistency requirements (other than the frame-invariance) imposed on the additive hypo-elasto-plastic split. One of the earliest and well-known consistency requirements is the Prager's yield stationarity criterion [55,56]. This criterion brings about the preference of the corotational objective stress rate formulation [57]. Consequently, corotational objective stress rates, such as the Jaumann rate and the Green–Naghdi rate, have been widely adopted in the additive hypo-elasto-plasticity formulations. Some abnormal responses exhibited by these models, however, suggested for the need of additional consistency conditions to further narrow down the choice of objective stress rates. These abnormalities include the oscillatory stress response in the simple shear problem in the context of the Jaumann rate ([58–60]; see [61,62] for a remedy combining the Jaumann rate and hyperelastic relation) and the non-physical energy dissipation during pure elastic deformation [63,64]. As a remedy, Bruhns et al. [44] proposed a self-consistency condition stating that an additive hypo-elasto-plasticity model should produce a dissipation-free hyperelastic response in the absence of plastic flow. The corotational logarithmic rate [65–68] has been accordingly proposed since a hypoelastic model with constant material stiffness can be integrated to obtain a hyperelastic model provided that the logarithmic rate is employed [68,69]. Numerous elastoplastic constitutive models have consequently been developed based on the logarithmic rate ([44,70–76] among others). The self-consistency condition proposed by Bruhns et al. [44], however, may not be completely fulfilled with the logarithmic rate employed in the additive hypo-elasto-plasticity models. It has been recently shown in the so-called unloading stress ratcheting obstacle test [77] that non-physical energy dissipation still exists when material is subjected to elastic unloading following yielding. It has been proved that a simple modification made to the logarithmic rate formulation, coined as the kinetic logarithmic rate [77], fully satisfies the self-consistency requirement.

The additive hypo-elasto-plasticity models and multiplicative hyper-elasto-plasticity models are usually known as two distinct categories of elastoplasticity models [78–81] and several comparative studies have been conducted to investigate the difference between the two [82–84]. In these studies, several specific objective stress rates are employed in the additive hypo-elasto-plasticity models.

In the present manuscript, it is shown that a typical multiplicative hyper-elasto-plasticity model for isotropic materials coincides with an additive hypo-elasto-plasticity model based on the modified kinetic logarithmic stress rate employed in the hypoelastic relation. In the absence of strain-induced anisotropy characterized by kinematic hardening herein, the modified kinetic logarithmic stress rate reduces to the kinetic logarithmic stress rate recently proposed in [77].

The manuscript is organized as follows. The basic kinematic definitions and relations are introduced in Section 2. In Section 3, a summary of typical structures of multiplicative hyper-elasto-plasticity and additive hypo-elasto-plasticity