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Adaptive stopping criterion for iterative linear solvers combined with anisotropic mesh adaptation, application to convection-dominated problems

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Abstract

In this work we propose an automatic adaptive stopping criterion integrated with the anisotropic mesh adaptation procedure, thus requiring no additional computational cost. Using information from this procedure we provide control for the linear solver convergence, stopping the iterative solution when the algebraic error is lower than the estimated discretization error. We apply this framework to steady and unsteady convection–diffusion problems, using a stabilized finite element formulation. The proposed method proves to be an effective cost-free strategy to reduce the number of iterations needed, without spoiling the accuracy of the solution.

Keywords: CFD; Stopping criterion; Anisotropic error estimator; Mesh adaptation; Unstructured meshes; Convection–diffusion

1. Introduction

Adaptive finite elements methods (AFEM) are nowadays well known to be a reliable approach to achieve better accuracy in the solution of PDEs, with a reduced computational cost. These approaches rely on error estimates, either *a priori* or *a posteriori*, to identify the regions of interest to be adapted. However, for CFD applications *a priori* error estimates are frequently unreliable, especially in the presence of singularities. Moreover, in many applications the solution is characterized by distinctive directional features, such as boundary or internal layers. In these cases an effective mesh adaptation technique must be able to provide the best suitable orientation of the elements. For this reason a directional *a posteriori* error estimator is required to drive the adaptation procedure. This subject has been addressed by several authors [1–4]. A possible choice is to rely on the recovered Hessian of the solution as a base to build the estimator [1–3]; the Hessian can be used also in combination with a PDE-dependent estimator to improve accuracy, as suggested in [4].

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A typical adaptation algorithm starts from a solution on an initial discretization of the computational domain, computes the estimated error distribution on that mesh, and using this information defines the features of the new spatial discretization. The solving step implies the solution of the linear system that stems from the discretization of the continuous problem. Usually the solution of this system is assumed to be exact. In real-world applications however, this kind of system can be solved efficiently only with an iterative procedure, which provides an approximation to the exact solution. The accuracy of this approximation is controlled by the stopping criteria used to drive the convergence of the iterative procedure.

As remarked by Becker et al. in their seminal work [5], ad hoc stopping criteria are commonly used, e.g., requiring an initial residual norm to be reduced by a certain factor. These criteria are straightforward to implement, but have no direct link to the actual error in the approximate solution. This could possibly affect the efficiency of the iterative procedure and the accuracy of the resulting solution. On one side a highly accurate approximation is inefficient and most likely unnecessary, on the other side a poor approximation affects the accuracy of the solution and the convergence of the adaptation procedure. This point of view has been developed in several works regarding inexact iterative solvers and stopping criteria. In the framework of symmetric problems, Arioli in [6] proposes an *a priori* stopping criterion and Picasso in [7] suggests an *a posteriori* approach using an AFEM algorithm. The method proposed in [8] is based on the use of a proper weighting of the residual vector as an estimator for the algebraic error, with applications to elliptic problems, using finite volumes and mixed finite elements. The same subject is addressed in [9] for nonlinear elliptic PDEs, using equilibrated flux reconstruction estimators. [10] discusses the convergence optimality of the AFEM algorithm using a stopping criterion for the iterative solver based on *a posteriori* error estimators. An interesting overview of the existing approaches is proposed by Arioli and co-workers in [11]. Many of these approaches rely on the property of the Conjugate Gradient (CG) method to minimize the energy norm of the discretization error at each iteration, as shown in [12]. However, the Generalized Minimal Residual method (GMRES), used to solve nonsymmetric problems, does not hold the same property and minimizes only the euclidean norm of the residual. In this work we propose an adaptive stopping criterion that follows the strategy developed for symmetric problems in [7], extending the application to non-symmetric problems. We use the general anisotropic mesh adaptation framework detailed in [13], that relies on a recovered Hessian based error estimator computed in the L^p norm and is shown to be robust and problem independent. Using information from the adaptation procedure we provide a cost-free automatic adaptive control for the linear solver, that proves to be effective to drastically reduce the number of iterations needed, without spoiling the accuracy of the solution. We extend the application to steady and unsteady convection–diffusion problems with high Peclet number, relying on a stabilized finite element method [14–16].

This paper is organized as follows. In Section 2 we introduce the convection–diffusion model problem and the relative stabilization technique, in Section 3 we present the error estimator and the procedure used to provide the metric for the mesh adaptation. In Section 4 we provide the adaptive stopping criterion to control the iterative solver, and finally in Section 5 we validate this framework with several test cases.

2. Convection–diffusion problem

In this section we introduce the general equation of the unsteady convection–diffusion equation.

We consider the following problem that models the transport of a quantity u through convection and diffusion

$$\begin{cases} \partial_t u(x,t) + \mathbf{v}\cdot\nabla u(x,t) - \nabla\cdot(k\nabla u(x,t)) = f & \text{in } \Omega\,, \\ u(.,0) = u_0 & \text{in } \Omega\,, \\ u(x,t) = g & \text{in } \Gamma\,, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^d$ is a bounded polyhedral domain with boundary Γ. For the sake of simplicity we consider Dirichlet boundary conditions. Here k is the diffusion coefficient, $\mathbf{v} \in \left[W^{1,\infty}(\Omega)\right]$ is the divergence-free velocity field, $f \in L^2(\Omega)$ is a given source term, u_0 is the initial data, and g is a given boundary condition.

2.1. Galerkin finite element formulation

We define a variational problem considering the functional Sobolev space of functions with square integrable first order derivatives $H^1(\Omega)$, where

$$H^1(\Omega) = \{w \in L^2(\Omega), \|\nabla w\| \in L^2(\Omega)\}\,, \tag{2}$$