Available online at www.sciencedirect.com
ScienceDirect

Comput. Methods Appl. Mech. Engrg. 339 (2018) 262–280

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

A stabilized non-ordinary state-based peridynamic model

P. Li[a], Z.M. Hao[a,b,*], W.Q. Zhen[a]

[a] *Institute of Systems Engineering, CAEP, Mianyang 621999, China*
[b] *Shock and Vibration of Engineering Materials and Structures Key Laboratory of Sichuan Province, Mianyang 621999, China*

Received 21 November 2017; received in revised form 26 April 2018; accepted 2 May 2018

Highlights

- A new stabilized non-ordinary state-based peridynamic model is proposed.
- The proposed model eliminates zero-energy modes control coefficient adjustment.
- The proposed model controls the zero-energy modes effectively.
- The parameter of strain energy ratio is defined.

Abstract

Non-ordinary state-based peridynamics suffers from zero-energy modes due to nodal integration, causing instabilities of the displacement, stress and strain fields. A stabilized non-ordinary state-based peridynamic model is derived according to the linearized bond-based peridynamic theory. The incorporation of supplemented force state excludes a zero-energy modes control coefficient and eliminates the need for complicated parameter adjustment. Finally, the parameter of strain energy ratio is defined to study the degree of influence of the zero-energy modes on the computation process. Four numerical examples are analyzed to demonstrate the effectiveness of the present model in controlling the zero-energy modes in non-ordinary state-based peridynamics.

Keywords: State-based peridynamics; Zero-energy modes; Stabilization; Damage

1. Introduction

Peridynamics (PD) is a meshless method that substitutes the governing partial differential equations in classical continuum mechanics with integral equations. It is valid regardless of discontinuities. Therefore, peridynamics is useful to solve problems involving spontaneously emerged discontinuities [1–3]. In the original formulation of peridynamics, usually referred to as the bond-based theory, the response of a bond is independent of the other bonds. As a consequence, the bond-based peridynamic scheme is limited to constitutive models with a fixed Poisson's ratio (1/4 for three-dimensional and plane strain problems, 1/3 for plane stress problem) [4–6]. Because bond-based theory does not distinguish between volumetric and deviatoric deformation, it is not suitable to capture the

---

[*] Corresponding author at: Institute of Systems Engineering, CAEP, Mianyang 621999, China.
E-mail address: haozm@caep.cn (Z.M. Hao).

https://doi.org/10.1016/j.cma.2018.05.002

plastic incompressibility condition [7]. To address these issues, Silling [8] developed the non-ordinary state-based peridynamics (NOSBPD) which makes it possible to incorporate general constitutive models and failure criteria. Recently, this approach has attracted more and more attention [9,10].

Nodal integration schemes that utilize a deformation gradient tensor to define strain are desirable due to their high computational efficiency and the absence of the background cell structure or background mesh employed for Gauss quadrature. However, they lead to zero-energy modes and tend to deform infinitely under a perturbation without energy consumption. Similar to Smoothed Particle Hydrodynamics (SPH) and meshfree Galerkin method, NOSBPD use of nodal integration also experiences zero-energy modes [11–13]. The oscillations of the displacement, stress and strain fields are induced. They can affect the computational precision or eventually ruin the results. Several control methods of zero-energy modes have been developed for non-ordinary state-based peridynamic analysis. Littlewood [14] used a penalty term added to the force state, where the penalty force is proportional to the difference between the actual position of a point in the deformed configuration and the position predicted by the deformation gradient. Breitenfeld [15,16] added supplemental interconnected springs between a particle and all the particles belonging to its horizon or a zero-energy modes control term based on the averaged displacement over all the particles in the horizon to the original force state. Nevertheless, a common feature of these zero-energy modes control methods is an involvement of a stabilization parameter that is material and peridynamic grid spacing sensitive. If the parameter is too small, the zero-energy modes cannot be effectively suppressed. But for very high values of the parameter, the correction term starts to dominate the solution, easily leading to non-convergence of the solution. Thus, the parameter value is of great significance and needs to be adjusted properly especially when the nonlinear constitutive model is considered. Wu and Ren [17] introduced a stabilized displacement field to control the zero-energy modes. The stabilized displacement of each particle is determined by averaging displacements of all its neighboring particles. Becker and Lucas [18] replaced the velocity of a node with a weighted sum of the original velocity and the average velocity of all nodes in the family, and produced a reasonably steady velocity field. The methods provided by Wu and Ren [17] or Becker and Lucas [18] are smoothing techniques and do not specifically target the zero-energy modes. They also alter aspects of the solution not related to parameter fluctuation modes. Although these methods are devoid of supplemented force state, the oscillation problem appears to remain in stress and strain fields. The zero-energy modes in non-ordinary state-based peridynamics are mainly due to the calculation error of the approximate deformation gradient tensor. Hence, the oscillations can be reduced by improving the computational accuracy of the approximate deformation gradient. Yaghoobi and Chorzepa [19] improved the precision of the deformation gradient tensor from first-order to second-order, fourth-order, or even sixth-order by fixing the influence function of particles. The instability in displacement solutions, as well as the noise in stress and strain fields, is decreased. Silling [20] derived a stabilized correspondence material model through the minimum potential energy principle. He investigated the phenomenon of zero-energy modes as a material instability rather than a numerical instability. This approach provides a good way for zero-energy modes control in NOSBPD. Nevertheless, the control coefficient needs to be determined by a large number of numerical experiments. That consumes plenty of computation time, and the failure mode of the plate with a circular hole given in paper [20] may not be optimal.

The purpose of this paper is to present a stabilized non-ordinary state-based peridynamic model in which the numerical instability due to zero-energy modes is suppressed. The outline of this paper is as follows. The non-ordinary state-based peridynamic formulation is first reviewed in Section 2. A stabilized non-ordinary state-based peridynamic model and its derivation process are described in Section 3. Four numerical examples are presented in Section 4 to demonstrate the accuracy and effectiveness of the proposed model in controlling the zero-energy modes in NOSBPD. Final remarks are drawn in Section 5.

2. Review of non-ordinary state-based peridynamics

2.1. Equation of motion

The peridynamic equations of motion are given by [8]

$$\rho\ddot{\mathbf{u}}(\mathbf{x},t)=\int_H\left\{\underline{\mathbf{T}}[\mathbf{x},t]\left\langle\mathbf{x}'-\mathbf{x}\right\rangle-\underline{\mathbf{T}}[\mathbf{x}',t]\left\langle\mathbf{x}-\mathbf{x}'\right\rangle\right\}\mathrm{d}V_{\mathbf{x}'}+\mathbf{b}(\mathbf{x},t),\tag{1}$$

where ρ is the density, $\ddot{\mathbf{u}}$ is acceleration, $\mathrm{d}V_{\mathbf{x}'}$ is the volume associated with material point $\mathbf{x}'$ in the undeformed configuration, $\mathbf{b}$ is an external body force density field, and $\underline{\mathbf{T}}[\mathbf{x},t]\left\langle\mathbf{x}'-\mathbf{x}\right\rangle$ represents the force state exerting on point